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Real Challenges in Mathematics Learning

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Preface

The College of Natural Sciences and Mathematics of the University of Central Arkansas was pleased to host the 37th annual meeting of the Research Council on Mathematics Learning (RCML). The theme of this year's conference, *Real Challenges in Mathematics Learning,* provided participants an opportunity to explore a wide variety of topics including mathematics learning for students for whom English is a second language, the impact of demographic composition of a school on comprehensive testing assessment and accountability, and how non-traditional instructional devices such as movies, literature and the internet promote critical mathematical fluency.

The collegiality of the 80 plus conference participations representing colleges and universities in 15 states exemplifies the importance for a professional exchange of ideas and the opportunity for dialogue with the speakers and invited lectures.

Conference planners and the RCML leadership opted for an added dimension as a follow-up to the 37th year's program; a conference proceedings document that would provide yet another way for documenting the outstanding work of presenters and establishing another opportunity for professional dialogue with conference participants and beyond. Prior to the conference, participants were invited to submit manuscripts of research findings to be presented as part of the conference. Seventeen manuscripts were submitted for review. A technical review committee chose thirteen papers for inclusion in this the first annual Conference Proceedings. These proceedings are offered as a new venue for publishing primary work in research in mathematics learning.

The local organizers for the conference are indebted to the faculty and staff of the University Of Central Arkansas Department Of Mathematics and to the staff of the Arkansas Center for Mathematics and Science Education for assisting in so many ways with the conference details.

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RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

Appreciation

We would like to thank the following for their support and sponsorship of this meeting. The Arkansas Center for Mathematics and Science Education and the Department of Mathematics at the University of Central Arkansas for hosting this conference. The faculty, staff, students, and participants were instrumental in the success of this conference.

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HOW DEMOGRAPHICS AFFECT MATHEMATICS SCORES ON THE ACTAAP: 2007 SNAPSHOT

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The purpose of this study is to examine how demographic characteristics affect mathematics scores on the Arkansas Comprehensive Testing, Assessment and Accountability Program (ACTAAP). The study examined the relationship between the total percentage of students who scored proficient or advanced and the following demographics: gender, average daily attendance, millage, number of students eligible free lunch, number of students considered gifted and talented, Asian, Black, Hispanic, Native and White. The study focused on 40 school districts (in different regions of the state of Arkansas) divided into three sub groups: district size less than 750, district size ranging between 750 and 2000 and district size greater than 2000. Using the statistical software, MINITAB, data were analyzed by applying correlations, linear regression and analysis of variance (ANOVA). The results of the study suggest that the prevalent cause of low test scores in the state of Arkansas may be the high number of students qualifying for free lunch and the low millage rates, corresponding to previous research in this field.

Background of Study

In the state of Arkansas, assessment focuses on curriculum and testing. The *Arkansas Comprehensive Testing, Assessment and Accountability Program* (ACTAAP) encompasses the state's “Smart Start Initiative” for Grades K-4, the “Smart Step Initiative” for Grades 5-8; and the mathematics and the “Smart Future” of Grades 9-12. The *Smart Start Initiative* and the *Smart Step Initiative* are very similar programs implemented to improve student achievement. The programs focus on standards, professional development of teachers and administrators, student assessment and school accountability. ACTAAP focuses on accountability with an emphasis on well-defined, high educational standards in reading, writing and mathematics. It is designed to improve student learning and classroom instruction; provide accountability by establishing expected achievement levels and reporting student achievement; provide program evaluation data; and assist policymakers in the decision-making process (State of Arkansas. (n.d.))

The state of Arkansas is comprised of approximately 245 public school districts of various sizes. Students in third grade through the eighth grade in every district are required to take benchmark exams in literacy and mathematics. The test results are categorized as: below basic (students who fail to show mastery), basic (students showing substantial skills but who need additional assistance), proficient (students demonstrating solid academic performance and are well-prepared for the next level of schooling) and advanced (students who have demonstrated superior performance well beyond the proficient level of performance). In the study of test results, demographics may be important in predicting performance scores on the mathematics portion of the ACTAAP test. If there are known demographic factors, steps can be taken to help students excel on the test. Some of the factors are, but not limited to, absenteeism, poverty and non-trained teachers.

In this setting, a detailed study was conducted using the following demographics: gender, average daily attendance (ADA-4), millage, number of students considered for free and reduced lunch, number of students considered gifted and talented, and ethnicity. The school districts studied herein have been divided into three categories: school districts less than 750, those between 750 and 2000, and those greater than 2000. School districts in communities across the state of Arkansas are represented in each category. The data were collected from the Arkansas Department of Education website and are presumed accurate. By analyzing the relationship between demographics and scores from the 2007 data, correlations were determined between students' performance on the benchmark exams and the demographics studied. The results reveal connections between some of the demographics and the cause of the low test scores on the ACTAAP.

Review of Literature

Payne and Biddle (1999) studied the effect of poor school funding and poverty on mathematics achievement. Public schools in the United States are funded mostly by local taxes in each individual school district. Thus, funding in impoverished areas varies widely from funding in affluent communities. However, in many other countries, funding for education comes from the government and is equal amongst all students. This sometimes huge disparity in funding would seem to make a difference in student success in the classroom; but researchers have found very little correlation between the two factors according to Payne and Biddle. Hanushek, an economist in agreement, found an insignificant effect of funding on student achievement (Hanushek, 1989b). Greenwald, Hedges and Laine (1994a, 1994b) disagreed with Payne and Biddle (1999). They conducted a wide array of studies over three decades and showed that school funding affected student outcomes depending on where and how the money was applied. They stated that "school resources are systematically related to student achievement and that these relations are large enough to be educationally important (Greenwald et al., 1996)".

Payne and Biddle (1999) suggest that child poverty is a significant factor that affects student achievement in the United States. Children living in poverty face problems in the home which prevent them from focusing on school. Distractions include disease and sickness caused by a lack of health insurance, crimes in the neighborhoods, gangs and drugs. Furthermore, they may not have a parent that can help with their school work. According to Payne and Biddle (1999), these children also tend to lack access to books, stationeries, computers and other school supplies that are vital to success in the classroom. Also, students living in poverty are likely to attend schools with poor funding. Payne and Biddle concluded that child poverty and poor school funding have an effect on student performance on tests. However, the effects are largely independent of one another and they are also largely independent of factors such as race/ethnicity and the school curriculum (Payne & Biddle, 1999).

McDermott, Raley and Seyer-Ochi (2009) also agree that students' low achievement is correlated with poverty. However, they state that "limiting public knowledge of the poor to school performances hides their capacities and achievements and allows the lazy assumption that the absence of middle-class things (such as books, computers and tutors) degrades intelligent thought." Students who are poor and excelling in school are usually not discussed and students who are rich and doing poorly in school also go undetected in studies. In studying the effect of race and class, McDermott et al (2009) define "*race* as a trait given at birth and turned into trouble by prejudice and unequal conditions; *class* as traits socialized into children with diminished socioeconomic opportunities and *risk* as a result of children being damaged by racism and class disadvantage." The authors suggest that class and race should be considered a "social activity" and not how people are defined. Everyone in a society is responsible for finding a solution to the effects of race and class on education. Individuals belonging to either group should not be ignored and discounted because they are considered at risk as a result of the stigma caused by society. At-risk status has been wrongly characterized as an internal trait or innate characteristic and is, "rapidly becoming synonymous with 'minority' status as opposed to children being placed in the situation (O'Connor, Hill & Robinson, 2009)". Goldsmith's (2004) study demonstrated that minorities, specifically Hispanics and Blacks, fared better and were more optimistic about school when the school they attended employed many minority teachers. In this same environment, the study also showed that the Black-White and Hispanic-White gap in student achievement was reduced (Goldsmith, 2004).

Researchers have found that a wide discrepancy between White-Hispanic and White-Black scores is largely because of dropout rates between both groups. The reason for the high dropout rates can also be linked to socioeconomic status. The difference in achievement between racial groups also differs vastly from different states and different school districts across the country. Asian Americans have been shown to do better than whites, especially because of the value they place on education and the ``high educational expectations and effective child-rearing practices on the part of their parents (O'Connor et al., 2009)"

The neighborhood where a student lives also plays a role in performance at school. In turn, race and class may affect where a student lives. A neighborhood that is safe and where students have access to libraries and museums tends to stimulate a student's mind and provides a better environment for studying and focusing on school. On the other hand, neighborhoods that lack these facilities are usually filled with drug dealers, liquor stores, drive-by shootings and are typically not conducive to focusing on academics. McDermott et al. (2009), once again state that everyone from housing developers to the media is involved in creating the situation in which these students find themselves. They concluded that race and class have an effect on school performance. However it is ``the result of being put down, pushed down, cut off, and certified as failures (McDermott et al., 2009)"

In summary, previous studies have supported the idea that ethnicity, class, poverty, and poor school funding are demographics that are linked to student performance. With the results of these former studies in mind, the study presented here provides further evidence to support the relationship between certain demographics and large-scale testing results.

Methodology

Based on previous research, the study seeks to affirm or disagree with the statements below:

- The high number of students considered for free/reduced lunch and the low millage rate of the school district are statistically significant factors that influence mathematics test scores, and;
- The ethnicity of the student is a statistically significant factor in predicting mathematics scores on large scale assessment tests in Arkansas.

The data are analyzed using statistical software called MINITAB, which generated a regression analysis table to determine the demographics insignificant to each model utilized in the study. Models are determined based on the highest R-square and lowest PRESS across various grades in the three categories. The study began by investigating which regressors are highly correlated with each other. For each school district size, it was shown that gender and ADA-4 are very highly correlated with each other and thus were removed from the study. For the school districts less than 750 in size, the correlation between genders was 0.917, between male and ADA-4 was 0.977 and between female and ADA-4 was 0.972. The Stephens school district was removed in this school range because the ADA-4 was too high and skewed the model. For the school districts ranging between 750 and 2000 in size, the correlation between gender was 0.947, between male and ADA-4 was 0.993 and between female and ADA-4 was 0.960. Warren school district was also removed because the ADA-4 was too high and skewed the model. Finally, for the school districts greater than 2000, the correlation between genders was 0.999, between male and ADA-4 was 0.994 and between female and ADA-4 was 0.989. Removing the three regressors, the Variance inflation factor (VIF) was reduced from very high numbers to low numbers below 10.

MINITAB was then used to determine the best subsets for each grade level. To determine the best subsets however, the PRESS had to be low compared to the other subsets, the VIF had to be below 10 in the particular grade level, the *p*-value had to be below 0.2 and the R-square had to be high. A regression analysis was performed with the total sum of proficient and advanced serving as the *Response* variable and each set of subsets serving as the *Predictor* variables. The *p*-values for each individual regressor in each subset also had to be below 0.20 for the subset to be ideal. The remaining demographics were the factors that contributed to the percentage of students whose benchmark results are considered proficient and advanced. The best subset of each grade's study (within each sub group) will be shown in a table. A “four in one” graph showing a histogram, normal plot of residuals, residuals versus fits and residuals versus order is included for each result.

The “extra sums of squares” method was used to ensure that the removal of predictors was ineffective. This method uses the information produced in the analysis of variance given by MINITAB (Ryan, 2007). Finally, the coefficient of each predictor in the selected best subset was examined to determine if it impacts the scores positively or negatively. In summary, data analysis was performed for each grade level 3 - 8 in each district category. The results are presented in the next section with special attention to ethnicity, free and reduced lunch, school district millage rate and students considered to be gifted or talented.

Result and Discussion

The results indicated that the four most dominant predictors (demographics) are Millage, F/R Lunch, Black and White. These demographics mean the statements given in the previous section are valid and accurate. The coefficient of each of these predictors was examined to determine its negative or positive influence on the ACTAAP scores. Examining the coefficients of the predictor “F/R Lunch” indicates that the values are all negative. The negative coefficient means the high number of students being considered for free/reduced lunch negatively affects the scores. Students being considered for free/reduced lunch usually indicate a high poverty level. The results also support previous research conducted by scholars such as Biddle, Payne et al. mentioned earlier; child poverty and poor school funding have an effect on student performance on tests (Payne and Biddle, 1999).

The coefficient for millage was negative in the “less than 750” and the “750 – 2000” range. However, it was positive in the “greater than 2000” range. This result means that low district tax rates are more likely to affect test scores in smaller school districts as opposed to larger school districts. The average millage rate amongst the school districts increased slightly across the three subgroups. Therefore, it can be inferred that a higher millage rate does in fact affect student achievement in the classroom. The effect of ethnicity is also important in the results of this study. The coefficient of ``white'' was almost all positive except for “8th grade - less than 250” region. This result is not surprising because the Caucasian ethnicity is the most dominant ethnicity in the state of Arkansas. There is also a high percentage of Caucasians (compared to the other ethnicities included in this study) in the more affluent regions of the state. The percentage of Caucasians is given for the following school districts: 80% in Bentonville, 98% in Mountain Home, 93% in Greenwood, 93% in Huntsville, 97% in Paragould, 98% in Greene County Tech to name a few.

These districts are in cities considered more affluent than others in the state. The other ethnicities included in this study did not have an influence on the scores and had almost all negative coefficients. This result indicates that the Caucasian ethnicity has the greatest effect on the scores and the cause for the low scores is due to other socio economic factors in the students' background. The effect of students considered to be gifted and talented was also very minimal. The coefficients were all negative because the number of students considered gifted and talented was very low in comparison to the total number of students in the school districts. The district size had some impact on the scores. Turner stated that "schools in districts with more resources should have achievement levels at least as large as students attending schools in districts with fewer resources, if all other characteristics of districts were the same (Turner, 2000). This statement might not always be considered accurate in every circumstance but the results agree with Turner based on the scores.

In summary, the results of this study demonstrate that alleviating poverty may increase the scores on the ACTAAP exam. It also suggests that increasing the millage rate in smaller school districts could possibly produce an increase in student scores. As Greenwald et al. suggested, an increase in millage would only affect student scores if the money is applied to school resources effectively. The picture portrayed by the results of this 2007 snapshot is that in Arkansas for grades 3-8, the high number of students considered for free/reduced lunch and the low millage rate of school districts are statistically significant factors in predicting performance on the mathematics portion of the ACTAAP tests. These results suggest that the lower the millage rate and the higher the students considered for free/reduced lunch, the lower the scores on the exam. Furthermore, the results of this study portray ethnicity as a statistically significant factor in predicting mathematics test scores on the ACTAAP.

Implication for Future Research

The *Arkansas Comprehensive Testing, Assessment and Accountability Program* was created to ensure student achievement by measuring students' learning in the classroom. However, the results of this study indicate that student scores are highly influenced by poverty level and school funding. Therefore, the state of Arkansas should provide more resources for students living in poverty by providing funding in the form of feeding, housing, stationeries, computers and other resources that would help the poor students focus on their education. Future research to determine the effect of demographics on the mathematics ACTAAP could provide more specific results with the availability of more diverse demographic data. The inclusion of more schools in each school district level would also provide a more thorough analysis.

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A CASE STUDY OF REFORM MATHEMATICS CLASSROOMS IN A CHINESE SECONDARY SCHOOL

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This study describes mathematics classroom teaching and learning practices within a junior high school in a city of Southwest China where the teaching experiment took place. Classroom teaching and learning practices are primarily concerned with classroom organizations, interactions and social norms. The results indicate that a collective learning approach was taken in the classroom reform, in which mathematical communications and student engagement in classroom activities were promoted. However, mathematical learning still focuses on knowledge gaining and understanding rather than knowledge generation.

Introduction

Since the 1980s, National Council of Teachers of Mathematics (NCTM) has called for mathematics education in the US to shift its focus from gaining factual knowledge to understanding, communicating, reasoning and problem-solving, and developing individuals' dispositions (NCTM, 1989, 1991, 2000). It has advocated the classroom to be a legitimate learning community where learners develop sustainable and all-round abilities to meet the needs of the radically changing society (NCTM, 1991; NRC, 2001). However, the changes of teaching and learning in mathematics classrooms, as the real test of the implementation of reforms, have been difficult to render.

The calling for educational reform in the US is echoed globally. In the East, similarly, a comprehensive mathematics education reform movement has been underway throughout K-12 schools in China. The reform expectations are articulated in the Chinese new mathematics standards by Ministry of Education of the People's Republic of China (MEPRC, 2001), which particularly advocates the development of creativity and practical ability. Since the initial implementation of the reform in 2001, China has emphasized on seeking approaches to carry out the reform ideas in classroom practices. This study describes an attempt of mathematics classroom reform within a junior high school in a city of Southwest China, which is relative less developed area in China. The purpose of this study is to shed light on Chinese mathematics education reform and to provide reference for mathematics classroom reform in the US.

Theoretical perspectives

This research is grounded on socio-cultural and learning community theories. From socio-cultural perspectives, research on classroom teaching practices should address social contexts and interaction patterns as key aspects (Vygotsky, 1978). Theories of the learning community suggest that classrooms are social learning systems. To understand classroom practices is to understand the relationships of its components in the context of a whole (Bielaczyc & Collins, 1999).

Class norms and learning goals

Social norms (Wood, 1998) and socio-mathematical norms (Yackel & Cobb, 1996) play important roles in regulating the teachers' and students' behaviors in mathematics activities in the classroom. Social norms reflect the dynamic relationship between individuals' learning and social contexts. Socio-mathematical norms are especially relevant to the development of students' mathematical thinking and autonomy in learning activities

(Yackel & Cobb,1996). Learning community theories, on the other hand, describe classroom norms from the perspective of the whole learning community. As Bielaczyc & Collins (1999) pointed out, learning goals for a learning community play a vital role in developing the individual knowledge and skills through the advancement of collective knowledge and skills.

Interaction patterns and dynamics of interactions

Wertsch and Toma (1995) identified univocal and dialogical interaction patterns as ways to study the nature of classroom interactions. Brendenfur and Frykholm (2000) further suggested interaction patterns as univocal, contributive, reflective and instructive communications. In univocal interactions, the teacher delivers knowledge to students, and ensures students to receive them. In contributive discussions, students have opportunities to share ideas with each other. In both univocal and contributive conversations, the objectives of discussions are to help students acquire certain predetermined information and knowledge rather than to expand students' understanding based on their own ideas (Lloyd, 2008; Brendenfur & Frykholm, 2000; Wertsch & Toma, 1995). In reflective discussions, students not only explain and share their reasoning processes but also they make adjustments and generate a new understanding of their thinking by building on interactions (Brendenfur & Frykholm, 2000). The purpose of instructive communications is not only to generate new meaning from students' utterances but also to lead led the modifications of later instruction. From community learning theories, interactions function as "formulating and exchanging ideas" (Bielaczyc & Collins, 1999, p. 276). Dynamics and diversity are driving forces of meaningful interactions. Any small changes in utterances may result in dramatically different outcomes. Teaching is to create learning possibilities through interactions (Davis & Simmt, 2003).

Methodology

This study focuses on classroom organizations, interactions, and social norms. Classroom organizations refer to organizational structures of classrooms and arrangement of instructional activities. Classroom interactions are concerned with the nature of interactions and relationships between the whole classroom and small group discussions. Social norms are manifested through the interplay of classroom discourses and instructional organizations. Qualitative methods were used to identify patterns related to the focuses of classroom practices in the study (Miles & Huberman, 1994). Data collection included multiple resources: classroom observations, interviews, student work, and surveys. The researcher observed 13 lessons in three classes with three teachers for about one week in the school including 12 normal lessons and one exemplary lesson that was given by one of the teachers to the principals of all the middle schools in the district.

Results

Classroom organizations

Small groups were the basic functional units for in-class and out-class activities in the school. The formations of small groups were relatively balanced with genders, learning abilities, and interests. Members in a small group were accountable for the growth of the whole group. Instructional activities included individual studies, small group discussions (20-25 minutes), small group demonstrations, and whole classroom discussions (20-25 minutes). Individual studies were often done at home before the class meeting.

Class norms and learning goals

The classes attempted to develop a common language of articulating thoughts. For example, the teachers often articulated their expectations for classroom activities and discussions. Then, In group presentations, each group would articulate what they aimed to do with problem by explaining what the problem's conditions were and what they needed to do to solve the problem. The class social norms primarily included emphases on sharing, reflections, and learning skills. One of popular socio-mathematical norms was simplicity and efficiency of problem solving. When students presented different approaches to solve a problem, the teachers compared the approaches and highlighted the simplest or the most efficient approach. The small groups had accountability for the learning of the groups. The classes conducted on-going evaluations on small groups. The criteria of evaluations on small groups were flexible and included multiple aspects of classroom and school activities so that each student in the group had a way to contribute to the group.

Interaction patterns and dynamics

The primary patterns of interactions observed in small groups are illustrated in Figure 1 – Figure 4. In Figure 1, the advanced student (ST1) played the role of a person who conveyed his/her solution to other students in the group. If a member (e.g. ST4 in Fig. 1) had question to the solution, the advanced student (e.g. ST1) would explain and re-deliver the solution to the group. Sometimes, two or three students in the group developed the solutions and then they explained to the rest of the group (Fig. 2). Figure 3 reflects contributive interaction in which the two students exchanged their thoughts to reach to a consensus on a solution or shared different approaches of solving the problem. The nature of discussions was primarily univocal and contributive. Students whose mathematics learning abilities were closer in the group tended to show contributive or dialogical interactions. However, dialogical interactions were very few in group interactions. The purpose of small group interactions focused on ensuring the members' understanding of how to solve the given problems and then they were able to present the solving process to the class.   In general, small group discussions functioned as a place for a group to reach solutions to the given problems and to ensure each member with an understanding of solutions.

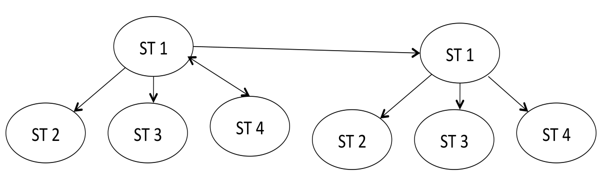

Figure 1

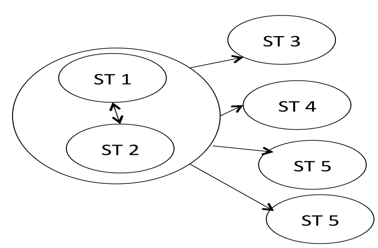

Figure 2

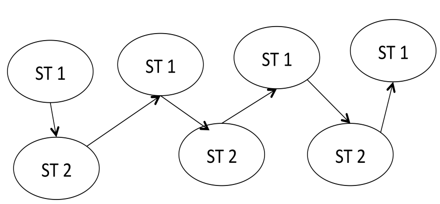

Figure 3

The whole classroom discussions were primarily contributive discussion which consistently revealed the following patters: Students presented and explained their solutions, and the teacher elicited key aspects of solving the problems; students presented and explained their solutions, and the teachers evoked alternatives and compared different approaches; students presented, explained, and reflected on their solving processes, and the teachers extended the understanding of knowledge and strategies. Univocal interactions occurred when the teachers wanted to emphasize the mastery of fundamental mathematical concepts and theorems. Reflective interactions occurred only when unexpected approaches appeared in the whole classroom discussions. Both univocal and reflective interactions were showed fewer times than contributive interactions. There were almost no instructive interactions. The whole classroom discussions appeared as a combination of contributive interactions with some univocal and few reflective interactions. Figure 4 shows the interaction patterns in the exemplary lesson given to the principals from all middle schools in the district, which can be viewed as a typical model of interactions in the whole classroom discussion. Over all, the whole classroom discussions functioned as a place where students demonstrated their gaining and understanding of the predetermined knowledge and skills and a place where the teachers monitored and regulated students' learning processes. Seeking strategies and skills to solve problems prevailed in the whole classroom discourses. It is obvious there were rare explorations and negotiations of mathematical ideas in the whole classroom discussion.

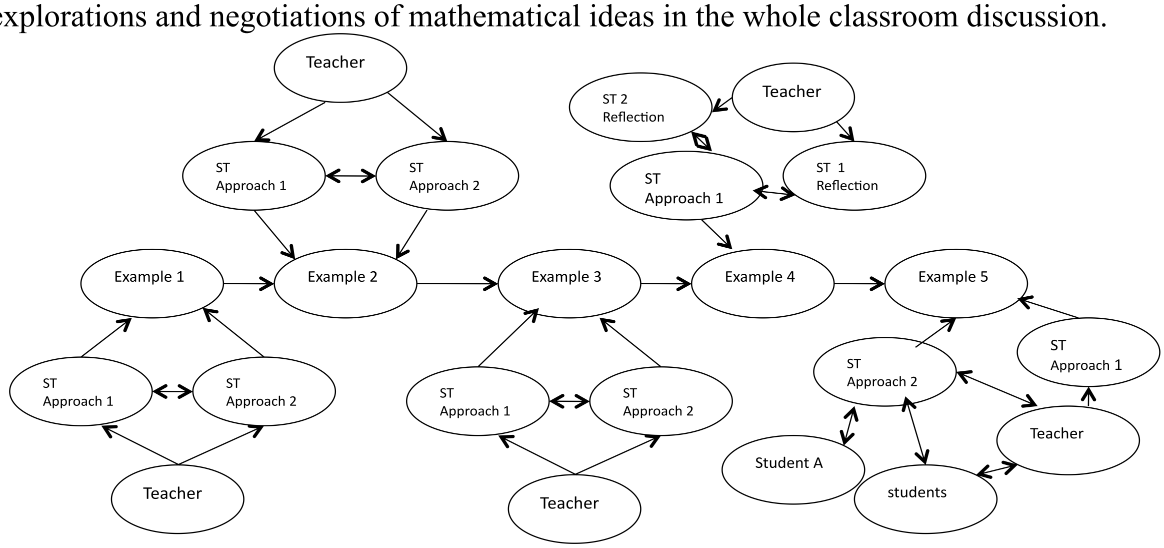

Figure 4

In both small groups and the whole class discussions, there were few occasions in which students had dynamic exchanges of ideas or ideas that were built off one another. Students shared alternatives, however, their struggles or failed attempts in solving a problem were ignored in most of situations.

Relationship between individual learning, small group learning, and the whole class discussions

It is evidenced that individual learning, small group discussions and whole classroom discourses enhanced and reinforced each other. In addition, teaching and learning reinforced and enhanced each other. In the presentation, students presented and explained their solutions to the class and reflected the key knowledge and strategies they used in their problem solving in which the students as a group played the role of a teacher to demonstrate and to guide the class learning. The teacher and other students listened to the small group presentation. Simultaneously, the teacher and others further elicited the key pointes and provided alternatives to extend understanding. The teacher and the class evaluated the small group performance. In both the small group and the whole classroom interactions, students seemed to play the roles of both students and the teacher.

Conclusions

The results of the study indicated a collective learning approach was taken in the classroom reform where students solved the given problems and shared understanding in small groups, and then demonstrated in the whole class. In the whole classroom demonstration and discussions, students in groups articulated their solutions to the problems and understanding of related mathematics knowledge while the teachers played the main role to monitor the problem-solving processes and to highlight difficulties and essential aspects of knowledge, skills, and strategies in lessons. It seemed that the interactions across small groups and the whole classroom, and the playing of teacher-student dual role enhanced and reinforced understanding and knowledge gaining. However, the dominated patterns of in both the small groups and the whole classroom discussions were contributive and univocal interactions. The lack of inquiry and argumentations in the interactions indicated that mathematical learning is still focused on the mastery of predetermined mathematical knowledge and skills rather than on exploration of mathematical ideas. Moreover, hardly any problem in the lessons involved context of real-life situations. The class norms emphasized perfectness and effectiveness of solutions rather than possibilities of emerging ideas. These aspects might diminish dynamic interactions and further limited the development of creativity and practical abilities.

Overall, the study revealed that changes occurred and issues of classroom practices accompanied with the reform approach in the school. On the one hand, this approach promoted mathematical communication and encouraged students, in particular lower level students to participate in learning activities; on the other hand, the nature of discourse has not changed. The study's findings imply the complexity of changing mathematical classroom practices, which requires a comprehensive consideration of classroom organizations, norms and interactions as a whole.

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LEARNING MATHEMATICS THROUGH PROBLEM SOLVING IN AN ON-LINE SETTING

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The purpose of this ongoing study is to explore the role of learning math through problem solving in an on-line math content course taught to first-year math teachers. The participants were enrolled in a Master of Arts in Teaching (MAT) program. The study was done in a three-credit math content course which is taught fully on-line. Course grades, math content assignments, and feedback from students were examined to determine the impact on learning math. Content was taught through problem solving in cooperative learning (CL) groups. When students did problem solving in CL groups they learned significantly more content, at the $p < 0.01$ level than when they worked in isolation.

Introduction

The logic behind learning mathematics through problem solving (PS) is that learners develop an understanding through reasoning and strategizing about solving real problems that require the use of mathematical ideas. Students learn how to apply concepts at the same time that they develop skill in doing mathematics (NCTM, 2000). In the US, we're working on this.

> Students need many more experiences in PS to apply and strengthen their mathematical abilities [including] the ability to reason, to communicate mathematically, to conjecture and test strategies that students are developing for solving problems, and to explore new and challenging problem situations without knowing exactly how they will solve the problem. (Sakshaug, Olson, and Olson, 2002, p. v)

It is challenging to teach mathematics through PS, for a variety of reasons, including teacher hesitancy, selecting good problems, and concerns about math anxiety. Suggest that a teacher teach through PS on-line, and the challenge has the potential to overwhelm to the point of surrender. Yet, if learning math through PS is as powerful as seen in research, (Lester, 2003; Sakshaug & Wohlhuter, in press, 2010) and the on-line setting is how many are taking math courses, it's necessary to explore the impact when the two are combined.

The question in this ongoing study is- what is the impact on learning when mathematics is taught through PS in an on-line math content course taught to first-year teachers? Students were engaged in PS with the goal of having them learn mathematics content (Lester, 2003; Van de Walle, 2004). Mathematical PS was implemented using cooperative learning (CL) in an on-line setting to teach number theory, algebra, geometry, probability, statistics, and logic.

A challenge that faced the teachers was that most had last been students years before the curriculum based on the *Principles and Standards for School Mathematics (2000)* was published. Teaching a curriculum which includes reasoning, problem solving, and applying mathematics is difficult if teachers aren't provided such experiences as learners (Ball, 1990; Herrera, 2005; Thompson, 1984). Content was selected to bridge potential content gaps. Teachers' experiences as problem solvers, their written metacognitive reflections, and the on-line discussions were designed to support them in learning and teaching via PS. (Ozsoy & Ataman, 2009).

Design

The study is grounded in the framework of knowledge as a social construction. The teacher-researcher is constructing understanding of the depth of learning of mathematics content as methodologies are explored related to engaging learners in problem solving in an on-line course (Dougiamas, 1998). This process constantly requires examination and adjustment of modes of teaching as they relate to feedback from learners. In addition, the learners are constructing their own understanding and adjusting their schemas. Such adjustment on the part of the learner is exhibited in a variety of ways, as the learner interprets learning and experiences (Lomax, 2000). DuFour's (2004) three big ideas that represent the core principles of learning communities were adapted to develop the on-line community in order to support social interaction. DuFour's principles are ensuring that students learn, creating a culture of collaboration, and focusing on results. The teachers needed to learn the content, to collaborate, and to focus on their own results as learners of mathematics and mathematical PS.

The data for this study was gathered from two years teaching the course. All math content was taught via problem solving. Teachers also reflected on readings about content, participated in on-line discussion, and wrote reflections on PS. Course grades, work on problem sets, and feedback via written reflections and discussion boards were used to determine the impact on learning. Each week of the term, teachers were given a set of six challenging problems to solve. The first week's topic was PS as a content area of math. The topics for each successive week were studied in the following order: number theory, algebra, geometry, probability, statistics, and logic. Topics were chosen because of the direct connection to the content the teachers would later be teaching, which they may not have had as undergraduates.

Teachers were to work on solving the problems on their own then bring their ideas to the CL groups for further discussion, exploration, reasoning, and solving. Individuals submitted solution processes to five of the six problems assigned, ten days after the assignment was given, including their own means of solving the problems they chose; how the process of working in the group went; the difficulty of the problem on a scale of 1-5 (1- an exercise, 5- a real problem); a description of the math they were doing; and a real-life application. The emphasis in the write-up and in grading was placed on process. The selection of good strategies was weighed heavily. This component of the course was 40% of the grade. Teachers were to communicate with their CL groups for feedback, suggestions and ideas.

In Year 1, there were six active students in the course, of nine enrolled. In Year 2, there were thirteen students in the course, twelve of whom were active. The PS tasks were arranged the same way in both terms and the same problems were used. The class was given the same process instructions both years. Teachers were instructed to work individually before collaborating in order to provide them with a chance to make sense of the problem and explore possible strategies on their own, otherwise the group tended to go along with the first idea shared. Moving quickly to one solution risked limiting to one idea rather than processing a range of ideas. Working individually first, more ideas and strategies were generated and shared. Different interpretations of what the problem was asking which provided opportunities for negotiation and positive interdependence (Johnson & Johnson, 2009).

Results

Six teachers participated in Year 1. There were 12 in Year 2. Course grades, work on assignments, and feedback in written reflections and on discussion boards were examined to determine the impact on learning. From Year 1 to Year 2, teachers had similar course grades. This didn't accurately reflect the grading plan designed. In Year 1, teachers ignored the CL requirement. Many were slow to get started in the course, so that assignments were coming in at very different times. This made it challenging to enforce forming groups and grading the group component. When teachers didn't form CL groups, the requirement wasn't imposed.

Comparing Year 1 to Year 2: Problem Solving Assignments

There was little review or reflection beyond first attempts in Year 1 since students didn't work together. Students turned in their best first try and waited for a grade. They would give their own best attempt at solving a problem and move on to the next problem. As a result, students' written reflections showed little evidence of a deepening understanding of content.

During Year 2, when groups were chosen by the instructor rather than allowing students to choose groups and CL was again required, there was a great deal of communication and collaboration, as seen in the write-ups and discussions. Teachers developed an understanding of the content that wasn't seen the year before. Because teachers in Year 1 weren't penalized for not doing the required CL, their grades were inflated compared to the Year-2 grades.

Table 1

Comparison of means by content area and t-test

Content Area	Year 1 Mean (n = 6)	Year 2 Mean (n = 12)
Problem Solving	7.8	8.4
Number Theory	7.6	9.6
Algebra	8	9.2
Geometry	7.6	10
Probability	9.4	10
t-test	p-value	0.01023**

In Table 1, the mean scores on content area written assignments were out of 10 points possible. In spite of the inflated grades given in Year 1, when a paired, t-test was run on the mean scores on each assignment, the results were highly significantly different at the $p < 0.01$ level. The likelihood that the difference in scores was due to chance was less than 1%.

Feedback from Teachers

There was little written feedback about PS from Year 1. Teachers commented that they thought the problems were too hard but they had given the solution process their best shot. There was more feedback about success in Year 2, via the group explorations. In a Year- 2 discussion post about working with others to solve problems, Laura said the following:

> Having a partner with a different answer pushed me to re-examine my own work multiple times [and] to look at the other person's work. If I was just working alone I would have assumed my answer was right since the solution process I used seems logically sound.

Because students in Year 1 engaged in little discussion during PS, the process was often ended when a first logical solution was found. A year 2 student, Kanye concluded the following:

> A student that has multiple representations of a concept in their toolbox, and [h]as generated the web of understanding that links all of them will be a much more powerful problem solver and effective learner.

The differences in the level of development were evident in the depth of work teachers from the two years submitted to illustrate how they solved problems. Teachers in Year 2 showed understandings of multiple representations. If their first attempts weren't fruitful, they understood why they didn't work because they engaged in discussions during the CL component.

Discussion

The main change in the course from Year 1 to Year 2 was that Year- 2 teachers actually did the required CL. In Year 1, all but one teacher worked in isolation, although CL was a requirement. The Year 1 group did poorly with respect to PS, even on the easier problems. There was little advancement beyond initial attempts. In Year 2, students did CL. They struggled during week one of eight due to lack of preparation for multiple collaborations. By week two they had adjusted in order to have the interactions needed to for CL. Teachers said they were doing more on their own to prepare for CL discussions because they didn't want to look 'dumb'. They also reported making more connections because of discussion.

Written feedback indicated a greater depth of learning in Year 2. There were more instances where connections were made to real life applications and to teachers' own teaching. In the discussion posts, teachers indicated that they enjoyed the problem solving, the collaboration, and learning new content. This was no comparable feedback in Year 1.

Other possible factors that might have had an impact on the study had to do with how well students knew each other or the instructor. The teachers taking the content course in Year 1 had never worked with the instructor before. All but two students in Year 2 had taken a course with the instructor prior to enrolling in the content course. This might have had an impact on their commitment to the process. Also, teachers were grouped regionally so many knew other members of their group. This might have also had an impact.

The results from Year 2 were markedly stronger than those from Year 1. Teachers learned more math content. The fact that Year-2 teachers did the required CL and discussion, which were the primary differences between the two groups, appeared to have an impact on the learning of the content. In addition, teachers indicated that the process was very positive for them. Further research on learning math through problem solving in an on-line setting is warranted. Specifically, the impact of CL on the learning outcomes is worth further exploration. In addition, further research on the building of the on-line community is suggested.

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A MIDDLE SCHOOL MATHEMATICS TEACHER'S CHANGES IN INSTRUCTIONAL PRACTICES

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Project IMPACT (Immersion in Mathematics Pedagogy, Application, Content, and Technology) was a three-year program funded by a state Math-Science Partnership Grant in the mid-south area. The case study of "Don", a participant in the Project, provides meaningful insight into the effects of a well-developed, long-term professional development program on teaching behaviors and attitudes.

Introduction

Project IMPACT (Immersion in Mathematics Pedagogy, Application, Content, and Technology) was a three-year program funded by a state Math-Science Partnership Grant in the mid-south area. The Project was designed to provide professional development for middle school mathematics teachers, primarily in grades 5-8. Participants originally applied in pairs to the Project; however, unavoidable circumstances led to the dissolution of some partnerships. Each year, participants attended a two-week summer institute. At the end of three years, participants had attended institutes in geometry and measurement, probability and statistics, and engineering and mathematics applications. In June 2009, a cohort of 20 middle school mathematics teachers completed Project IMPACT.

Literature Review

The most important instrument for change in student achievement in mathematics is with teaching itself (Glenn Commission, 2000). Effective teaching requires continual improvement. This improvement includes opportunities to reflect and refine instructional practice through collaboration with colleagues and extended time for change to occur. Reflection upon the success of these interactions yields information necessary to modify instruction (Friel & Bright, 2001; Schon, 1983; Sowder, 2007; Vacc & Bright, 1999).

Changing teacher practices at the classroom level is most likely to occur when professional learning is situated within or directly linked to the culture of the schools (Friel & Bright, 2001; Scribner, 1999), in a "situative perspective" (Putnam & Borko, 2000). Without support and guidance, teachers are not likely to make major changes in their teaching practices (Borko, Mayfield, Marion, Flexer, & Sumbo, 1997; Friel & Bright, 2001; Sowder, 2007). Professional development that includes "a focus on content; in-depth, active learning opportunities; links to high standards; opportunities for teachers to engage in leadership roles; extended duration; and the collective participation of groups of teachers from the same school grade, or department" is effective in supporting teacher change (U.S. Department of Education, 2002, p. 4). These research-based components of effective professional development were incorporated into the design of Project IMPACT.

Method

Prior to their first meeting in June 2006, teacher participants submitted a lesson plan that they considered to be successful from the recently completed academic year. Over the course of two years, participants developed and revised two additional lesson plans which they implemented and then reflected upon in their journals. From 2006 to 2009, participants submitted 15 Reflective Journal entries providing data about individual context, thinking, and events impacting their efforts throughout the Project. These data were the basis for three case studies, one of which is the focal point of this paper.

One researcher analyzed all data. Entries for each question were reviewed and summarized before reading or beginning analyses of entries for subsequent questions. During the first reading, preliminary notes were taken to summarize each entry and coded back to the particular journal. Entries were read a second time, and concurrently interpretative notes were made in the journal margins adjacent to relevant entries. On the third reading, appropriate adjustments and additions were made to the summaries and interpretations. At this stage, categories emerging from the journal text were identified, and the summary notes were reviewed and coded for the identified categories. Finally, the responses were summarized and interpreted in narrative.

Results and Discussion

The focus case is "Don". He was unusual among his Project peers because his journals were the longest and most introspective of the group. His case illustrates that a professional development program that is designed to occur over time, using intense workshops, cohort groups, collegial partners, opportunities for hands-on learning, and opportunities for real-world testing can positively impact practicing teachers. Here is Don's mindset upon beginning the Project, "I came to this academy thinking it was going to be a glorified in-service. In-service is important, but I go away from in-services thinking this is not going to change the way I teach" (Reflective Journal, July 2006).

The first Project experiences involved an intense two-week workshop of classes that required the participants to be a student, think about new ideas related to teaching and learning, struggle with concepts and activities, and participate in discussion. These factors were catalysts to Don's reassessing his own teaching. Don's reaction to the first set of workshops was specific:

> After attending and participating in this academy, I see areas of my teaching that I want to overhaul. I have never been very good at using manipulatives in my math class. Now that I have participated in learning about and using manipulatives, I see their importance and plan to involve my students more by using manipulatives. (Reflective Journal, July 2006)

His first step was to begin to identify what was missing from his teaching and to assert that he would embrace the idea of using manipulatives.

Collegial partnerships and cohort groups were heavily emphasized in the design of Project IMPACT. Teachers were required to apply in pairs, preferably from the same school. Don's recognition of the benefit of partnerships was almost immediate. He states, "My partner has already been a valuable resource, helping me through this academy. . . . Working with a partner can only help, not only in teaching but also in class management" (Reflective Journal, July 2006). That relationship continued to grow and develop as they planned the required unit. Don says:

> There was some really rich discussion about how we would pace the assignment and, most importantly, the manipulatives we used. We spent a great amount of time discussing how we were going to use some of our manipulatives and visuals. (Reflective Journal, February 2007)

As planning continues, Don describes their efforts:

> My partner and I communicate really well together. . . . We try to "talk" at least once a week. It may be just a quick exchange in the hall about where we are in the class or how we are progressing with our lessons. Other times we exchange e-mails about a particular web site or activity on the internet. Lately, we have been meeting after school to discuss our lesson as well as our unit. (Reflective Journal, May 2007)

In reflecting on the first year of IMPACT, Don expressed conscious understanding of the merit of collegial relationships including attitudes, understandings, friends, and support sources.

Activities that compel teachers to engage instructional methods and materials in the same way their K-12 students would can result in changed attitudes about how instruction can occur. This influence on Don is evident in the following:

> There were so many times throughout this academy that we had to learn the 'why' of something. For example, 'why' is the area of a rectangle [equal to], length times width, or 'why' is the area of a triangle, ½ base times height? I am so guilty of telling my students the formula and just practicing problems, not letting the students discover on their own. (Reflective Journal, July 2006)

Recognition is the first step toward change. At this point, whether Don would continue to incorporate the "why" into his teaching was not known, but he had begun to recognize the need.

Two years after the above statement, this attention on "why" continues to be a lasting concentration for Don, who reports:

> I am trying so hard to get away from the "here is the equation and here is how you use it" teaching. I am trying to get more toward the "let's see if we can discover the equation for the area" type of teaching. Participating in IMPACT has shown me the importance of letting students discover certain concepts and allow them to make their own connections. (Reflective Journal, July 2008)

But, it is not enough to know what should be done. Don has learned that teachers must know how to examine their own teaching. He notes:

> For me the most valuable methodology that we discussed in IMPACT was Japanese Lesson Study. . . . I look more critically at my own teaching. I have learned that there are no perfect lessons, but that every lesson needs to be looked at with a critical eye. All lessons can be tweaked. After I teach a lesson, I am examining myself asking, what could be done better, is there a better example that I can use, were connections made or missed? Lesson Study opened my eyes to the importance of a thorough critique of all lessons taught. (Reflective Journal, July 2008)

What is unstated in the comments above is that he now knows more about how to examine a lesson. He knows how to evaluate whether the lesson resulted in learning and why. Consequently, he can examine himself, critique the lesson, and improve it.

In addition to changing attitudes about how instruction can occur, activities that compel teachers to engage instructional methods and materials in the same way their K-12 students would can result in changed attitudes about how *learning* can occur. What does it mean to be a learner? Don did not think about this until Project IMPACT. Learning about Japanese Lesson Study contributed to Don's changed views about student thinking. He explains:

> The Lesson Study was valuable. . . . After reading and discussing Lesson Study, I see I need to think more about how students think [and] I need them to express their thinking. I want students helping their peers and students going to their peers for help. (Reflective Journal, July 2006)

One year later, Don was identifying additional ways his ideas about student learning and thinking continued to change because of Japanese Lesson Study:

> I have tried to question my students more about their learning and thinking. Instead of just accepting a right or wrong answer, I question my students on why or why not a problem is right or wrong. I want them to explain their thinking, which is something very difficult for a middle school student to do. I attribute the discussion and study of Japanese lesson study to my wanting to know how my students are thinking or learning. Before we got into the Japanese lesson study, I accepted a correct answer as [meaning] this person knows and understands the concepts being taught. Since we have studied lesson study, I realize that just getting the answer correct does not mean students understand the concepts. We have to look deeper, and I am trying to do that in my classroom. (Reflective Journal, July 2007)

Don's statements above reveal that his experience with Japanese Lesson Study resulted in his gaining two specific ideas about teaching: teachers need to know how students think, and teachers should question students about their thinking.

While the three two-week summer workshops with two intervening academic years associated with the Project were sufficient time to result in meaningful change in teacher thinking and instruction, some significant changes could be identified after a much shorter period of time. Don himself realized that his first year in the Project had had an effect on him. He explains:

> Reflecting on this first year of project IMPACT, this project has really made me refocus on how to teach math. I think of this project as a methods class because I am learning about activities and methods that help students learn math concepts. (Reflective Journal, May 2007)

After participating in the second year of workshops, Don's journals documented a transition from a focus on his teaching to a focus on his students' learning:

> The coursework at IMPACT is challenging but not ridiculous. The days are very long but rewarding. Some class activities and exercises really make me struggle. I appreciate the struggle because I am reminded that my students sometimes struggle with [learning] some of the math concepts I am trying to teach. (Reflective Journal, July 2007)

During the third summer of Project IMPACT workshops, July 2008, Don reflected on his 24 months of experience. He thought about his own professional growth and how it occurred, the applicability of particular elements of the workshop, and the changes to his classroom approach. His reflections reveal insights into how his teaching and thinking have transformed. He articulates these changes below:

> Even though this will be my 14th year teaching, I feel [like] a young math teacher. I began teaching 6th grade social studies and now find myself a 7th grade math teacher. Before I was involved in IMPACT I taught math like a first year teacher – glued to the text book teacher's guide. Project IMPACT has been like having three math methods classes. I [now] feel that I can move away from the book and use activities. (Reflective Journal, July, 2008)

One reason Don believes he can "move away from the book and use activities" is because he now has a conceptual understanding of what the act of teaching is supposed to accomplish. Teaching is always linked to learning. He asserts:

> Most energizing to me is the fact that I have so many ideas and ways to [use] them in my math classes. Because I have completed many of the same activities that my class will complete, I can anticipate where problems may occur, I can better anticipate where miscalculations may occur, and I can understand why students may not understand the concepts being taught. (Reflective Journal, July 2008)

In the above comments, Don described the advantages of learning-by-doing. Here, Don referred to actually doing activities in the Project that he might use with his students. This is why Don believes that he no longer must be bound to the teacher's textbook manual. He now understands what a learner needs in order to learn and what the teacher must do if the student will learn.

In summary, Don began Project IMPACT skeptical that he would learn anything of value. To his surprise, the Project was filled with elements Don could learn and use in his teaching. Not only that, Don became introspective about his teaching and his students' learning. Finally, he expressed considerable growth in teacher thought and teaching skill.

Don's case demonstrates how professional development can transform a skeptic into a very willing learner. It illustrates how a confident teacher who is competent, uses traditional methods and thought, and is minimally introspective can be transformed into a teacher who is analytically introspective and self-critical about his own instruction. Don's case exemplifies how a teacher can become analytical about instruction related to student learning and change his teaching behaviors as a result.

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STUDENT ACHIEVEMENT IN SOLVING MATH WORD PROBLEMS

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High school students' achievement in solving mathematical word problems continues to be low on the state, national, and international assessments. Based on the factors identified by previous research studies that influence word problem solving performance such as comprehension, representation, ability to connect mathematical concepts, and attitude, this study used a diagramming method to evaluate the achievement on and attitude toward math word problems of 172 grade 11 students from a New Jersey urban school. Preliminary analyses of the data from the study with a pre and posttest research design showed that the diagramming method improved achievement of both Hispanic English Language Learners (ELLs) and African American learners whose First Language is English (EFLLs), but significantly of ELLs. The students' achievement and attitude toward word problems showed unpredictable patterns.

Problem and Related Literature

During the last two decades, educators have faced the challenge of improving students' mathematical skills. The assessment results from the National Association of Educational Progress (NAEP, 2009), the Program for International Student Assessment (2007), and the Trends in Mathematics and Science Study (2003) show U.S. students' poor mathematics performance on problem solving and geometry. According to the NAEP, the average scores on a scale of 0-500 increased during 1990-2009 from 213 to 240 for fourth-grade and from 263 to 283 for eighth-grade students, but did not reach the proficiency levels of 249 and 299, respectively. The scenario of twelfth-grade students' performance is particularly gloomy. In the year 2000, they could correctly answer only 4% of the volume and surface area related geometry word problems and during 1990-2008 their average scale scores hovered around 300, i.e., below the proficiency level of 336 (NAEP, 2008; NCTM, 2004). Many problems used in the above assessments are word problems. Research on problem solving in mathematics has often used word problems (Kilpatrick, 1985). René Descartes' (1596-1650) *Discourse on Method* as explained by Schoenfeld (1987) provides mathematical methods for solving problems. From 1894 (National Education Association) until recently (National Mathematics Advisory Panel, 2008; *Rising Above The Gathering Storm,* 2007; U.S. Department of Education, 1983*), t*he committees on education have pointed out to the need for improving students' knowledge in the areas of fractions, powers and roots, geometry, and real-life application of mathematical concepts. It is hard to believe that students continue to falter on these topics when tested. U.S. students' mathematics performance in general and the prevalent 20% passing rate in the state mathematics test at the research site raised the research question for this study to investigate the effects of a 6-week diagramming instruction on students' achievement in math word problems. The diagramming method requires students to understand the vocabulary, construct labeled diagrams to represent the mathematics involved, and solve the problem using the diagrams.

Polya's (1945) *How to solve it*, a *landmark research* in problem solving led many researchers to develop theories and guidelines. The theories of comprehension (Kintsch & Greeno, 1985; Vergnaud, 1998), representation (Goldin & Kaput, 1996), and problem solving (Schoenfeld, 1985) helped develop the theoretical framework for this study. According to Kintsch and Greeno's, the lack of linguistic comprehension causes many obstructions in the context representation and solution of a word problem. Two factors related to the language of mathematics emerging from this theory are students' knowledge of the meaning and application of vocabulary and symbols which must be addressed in the teaching of word problems. The theory by Vergnaud suggests that the representation process, independent of the student's natural language, begins when the situation in a problem interacts with the related schemas and the comprehension improves if the problem details are depicted in the representation. Therefore, a mathematical representation of a problem is possible if the context and concept in the problem is recognized by using any language. According to Goldin and Kaput, the mental representation produced due to the interface of the concerned schemas can translate into an external representation such as a diagram. Thus, the theories on comprehension and representation imply that students' previously developed schemas must communicate effectively with new information from a problem for its correct representation. Finally, the theory on problem solving by Schoenfeld suggests that students need to know the mathematical procedures, draw diagrams and confidently choose methods to solve problems.

The conceptual framework for this study was guided by previous researchers' notion that the diagramming representation to solve word problems has a number of benefits including improved math scores, improved comprehension, overcoming difficulty, and improved attitude toward mathematics (Hutchinson, 1989; Simon, 1986; Toppel, 1997; Waters, 2004; Zawiaza & Gerber, 1993). These researchers have predominantly involved students with learning challenges or used problems requiring basic arithmetic operations.

Researchers have also studied students' other skills that influence word problem success. For example, reading problems aloud (DeCorte, Verschaffel, & De Winn, 1985) or silently (Davis-Dorsey, Ross, & Morrison, 1991), the rewording of word problems (Santiago, Orrantia, & Verschaffel, 2007), and the schema-based training (Jitendra et al., 2007; Xin, 2008) have shown to improve performance. Regarding linguistic background, Bernando (1999) argued that bilingual students (Filipino) perform better when word problems are written in their first language than when they are written in their second language (English), but Barwell (2005) found that it was not significant. Earlier, Quinn & Spencer (2001) showed that college men scored higher than women on standardized math tests, but more recent research (Solazzo, 2008) does not support the view that gender affects math achievement. Although most students in these studies were from middle schools, the findings have implications for this study.

Finally, Schoenfeld (1985) suggests that students' mathematical beliefs influence success in solving nonroutine problems. Attitude is the result of a firm belief about something. McLeod (1992) noted from previous research that attitudes include beliefs and correlate randomly with mathematical achievement. McLeod suggests that students' dislike about a math topic for a long period of time negatively influences their belief system and makes their responses to that topic involuntary "that can probably be measured through use of a questionnaire" (p. 581). It transpires from McLeod's research that a student who has developed a negative attitude toward geometry may avoid working on word problems that require geometric representation.

The three integrated units of the Instructional Module addressed the elements from the literature that influence word problem success. Unit A focused on the teaching of the vocabulary and symbols, Unit B for students to identify the objects, their mathematical relevance, and views for diagramming representation of the word problems, and Unit C for mathematical connections and solutions using the definition of words, geometric postulates and theorems, and formulas.

Methodology

This study was conducted in two consecutive school years, 2007–2009. The participants were five teachers and 172 grade 11 students enrolled in a Mathematics Applications course in a

Northern New Jersey inner city high school. During a 6-week intervention, both groups used the same curriculum. The two experimental teachers used the Instructional Module to deliver the diagramming instruction to 84 experimental students in 8 classes, while the three control teachers delivered the conventional instruction to 88 control students in 10 classes. The instruments for this study were a pre and a posttest, each containing eight math word problems of the New Jersey High School Proficiency Assessment (HSPA) and SAT type and an attitude survey with 10 items similar to a mathematics attitude test (Brown, Cronin, & McEntire, 1994). The math tests were found reliable based on the parallel-form reliability ($r = .73$) and test-retest reliability ($r = .86$) and valid based on the NJ mathematics standards for content and the NJ HSPA 2001 test as a criterion ($r = .57$). The study's factors were treatment (control, experiment), learnertype (ELL, EFLL), classtype (bilingual, mixed), and gender (Female, Male). The math tests were graded according to the NJ HSPA rubric and considered reliable based on the inter-rater reliability ($r = .95$). Students' difference scores in word problems from pretest to posttest, DIFF and difference scores in attitude, PostAttd - PreAttd and PostPostAttd - PreAttd, i.e., after feedback of math scores (see Figures 1 & 2) were analyzed. The experimental students' class assignments and the actual pre and posttests indicated their grasp of the diagramming method and use of diagrams.

Findings

A preliminary data analysis using an ANOVA for treatment vs. difference scores (DIFF) at $\alpha = .05$ showed significant improvement ($p < .001$) of the experimental group ($M = 3.29$, $SD = 3.44$) over the control group ($M = 0.32$, $SD = 3.40$). Tukey-Kramer's HSD multiple comparisons suggested that all the experimental ELLs and bilingual classtype students did significantly better than their peers in the control group (see Figures 3 & 4). However, the mixed classtype students in the two groups did not differ in their performance significantly. At the beginning of Unit A, 90% of the experimental ELLs and 50% of the EFLLs indicated trouble with vocabulary and symbols. After completing Unit A, the bilingual students' 342 responses to the Unit B problems showed 95% correctly identified objects, 33% dimensions, and 67% views. The mixed classes on the other hand showed 65% correct diagramming and labeling on their 222 responses to Unit B. For Unit C, students in the mixed classes had 46% correct answers for object identification, 45% for diagramming, and 44% for mathematical connections and solutions on 390 responses. In a secondary analysis, the difference in the number of diagrams from pretest to posttest and the DIFF values of the experimental group showed a positive correlation ($r = .61$). The preliminary analysis of students' attitudes before and after the intervention and after they received feedback on their math scores varied without any identifiable patterns (see Figure 2).

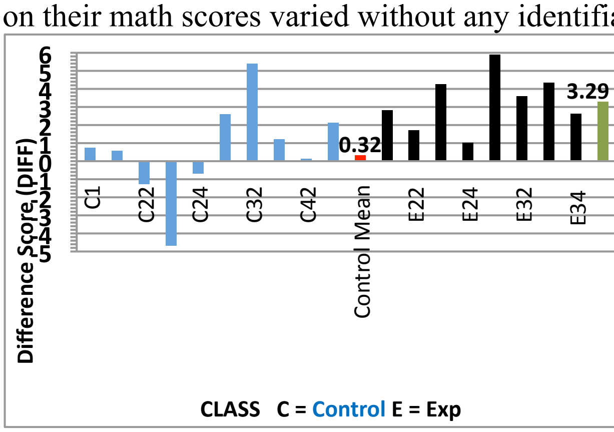


Figure 1. Student Achievement

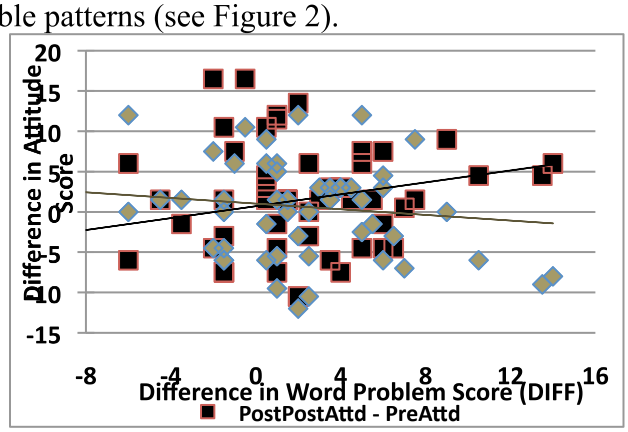
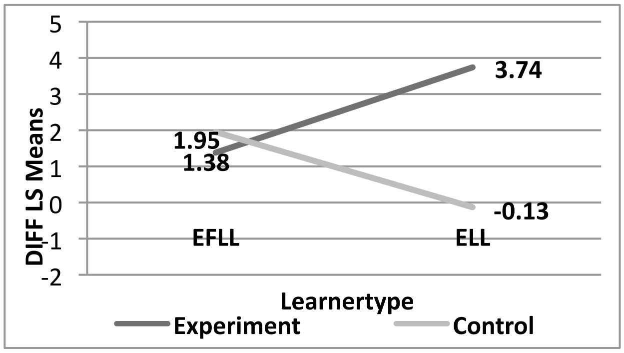


Figure 2. Attitude Toward Word Problems



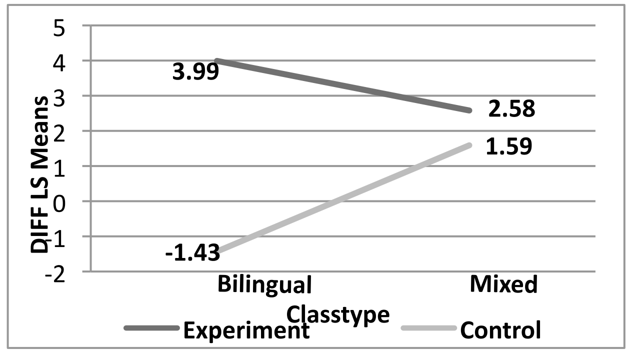


Figure 3. Treatment x Learnertype

Figure 4. Treatment x Classtype

Discussion and Implications

The data from this study indicated that students who used increasing number of diagrams could connect the underlying mathematical concepts and produce a greater number of correct solutions. Zawaiza & Gerber (1993) also found similar results such as increased precision in solutions after diagramming instruction. The Instructional Module in this study which is guided by schema theories was found to be effective for improving word problem solving skills of high school students, which supports similar findings by Hutchinson (1989), Jitendra et al. (2007), and Xin (2008). As identified by Simon (1986), the data from this study also indicated that with proper guidance students can draw meaningful diagrams. In other words, after appropriate schema construction, students can produce external representations from their internal representations as explained by Goldin and Kaput (1996). Contrary to the findings by Quinn and Spencer (2001), the results from this study showed that the word problem scores of male and female students' did not differ significantly. Also, contrary to the findings by Bernando (1999) that bilingual students (Filipino) perform better when problems are written in their first language, this study found significant improvement by the bilingual (Hispanic) students (see Figure 4) even when the word problems were written in English, especially after they understood the vocabulary and symbols in the problems. Thus, a possible way of improving student achievement is to focus on the teaching of vocabulary, symbols, and mathematical connections of contexts in problems using diagrams. Finally, students' mixed attitude toward word problems was evident, the reasons for which could not be explained from the study's data. However, their attitudes generally increased after they received feedback on their word problem scores (see Figure 2). Thus, a timely feedback may be considered as a motivating factor for students. Attitude evaluation through interviews and the use of diagrams to solve word problems at college merit future investigation.

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HOW DO CONTENT COURSES AFFECT CHANGES IN STUDENTS BELIEFS ABOUT TEACHING AND LEARNING GEOMETRY AS WELL AS INCREASING CONTENT KNOWLEDGE?

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Research has indicated that elementary teachers may not have the depth of mathematical content knowledge needed to teach mathematics successfully (Greenberg & Walsh, 2008). *Additionally, many teachers believe that the focus of mathematics is procedures and formulas with the goal of obtaining the correct answer (Mewborn & Cross, 2007). In an effort to better prepare pre-service teachers, curricula with problem solving activities and directed inquiry learning projects were used in mathematics content classes for elementary teachers. Content test and attitude surveys, both pre- and post, were administered to all the geometry classes for K-8 grade teachers to determine changes in both content knowledge of geometry as well as attitudinal changes about teaching and learning geometry. Does the content knowledge of our pre-service teachers increase and how do their attitudes about teaching and learning mathematics change?*

Related literature

Successful competition by the United States in the global economy depends on having adults who are well prepared in mathematics and science (National Center for Education Statistics [NCES], 2001). Adults of the twenty-first century need to be mathematically proficient in order to be productive members of our society (Ball, 2003) and the need for mathematics in everyday life has never been greater and indeed will continue to increase (National Council of Teachers of Mathematics [NCTM], 2000). Success in mathematics enables individuals to make choices concerning their futures and to be productive citizens (National Mathematics Advisory Panel, 2008). Mathematical competence can open doors to a productive future, while the doors to a productive future can remain closed for those students lacking in mathematical competence (NCTM, 2000).

As a nation however, the United States is not providing its students with the mathematical preparation needed to be successful. According to the results of the Third International Mathematics and Science Study (TIMSS), students in the United States achieve only at average levels when compared to students in other countries (NCES, 2003). In addition, according to the National Assessment of Educational Progress (NAEP), less than 20% of 12th grade students and about one-third of 8th grade students had achieved mathematical proficiency (Pehle, Wearne, Martin, Strutchens, & Warfield, 2004). Nationally, in 2009, only 34% of 8th grade students achieved proficiency and 27% were below basic. Therefore, even though mathematics scores on the NAEP have increased steadily since the 1970's, the gains are modest or even nonexistent (NCES, 2000). Furthermore, students in the state served by this study scored below average when compared to students across the United States, 33% of 8th graders in Georgia scored below basic in the 2009 assessment (NCES, 2009).

Teachers play a key role in ensuring that all students have the experiences needed to learn the mathematics necessary for success in future educational opportunities and careers (Mewborn, 2003). Teachers need a deep understanding of the mathematics they will teach (Conference Board of Mathematical Sciences [CBMS], 2001). Like students, teacher's content knowledge should include both procedural and conceptual knowledge with an understanding of how this knowledge is structured and generated throughout the domain of mathematics (Shulman, 1986). If students are expected to develop mathematical proficiency and to apply mathematics in real world situations, no less can be expected of their teachers (CBMS, 2001). However, the state of mathematics education in the United States does not promote the learning of mathematics in this manner. Teachers today, continue to teach as has been done for the last fifty years or more (Heibert, 2003). The Final Report (2008) of the National Mathematics Advisory Panel calls for substantial changes in the training of teachers if we are to do a better job of preparing students to compete in a global environment. Furthermore the United States cannot expect to be an international leader among industrialized nations in mathematics, science and technology unless these changes are made.

Teacher training institutions need to make substantial changes if teachers are expected to teach differently from what they have experienced. The Final Report (2008) reinforced the need for teachers to have a strong mathematics content of what they are to teach. Moreover, teachers cannot teach what they do not know (CBMS, 2001). The number of mathematics courses a teacher takes or the type of certification is not a good indicator of the content a teacher knows. Direct assessment, however, is the best indicator of teacher content knowledge and provides a determinant of student achievement (CBMS, 2001). As teacher content knowledge increases, classroom practices change. Teachers with strong content knowledge are more willing to try new ideas and rely less on the prescribed text (Mewborn, 2003).

Generally, elementary and middle school teachers do not have the conceptual underpinnings of the mathematics they know to be able to help students make sense of the mathematics as called for in the Principles and Standards for School Mathematics (Mewborn, 2003; NCTM, 2000). Memorization in place of understanding makes mathematics a set of disparate facts, procedures, and algorithms (CBMS, 2001) and can lead to an isolated skill set which can interfere with future attempts at sense making (Gravemeijer, & van Galen, 2003). Prospective and practicing teachers must be presented with the opportunity to learn and make sense of the mathematics that they will teach in order to instill in their students the confidence to make sense of the mathematics they are learning (CBMS, 2001). Research indicates that instructional practices aligned with NCTM Standards are effective but it is not easy to learn these practices nor has it been easy to obtain the training needed to teach according to these standards (Heibert, 2003).

Mathematics is a discipline of highly interconnected and integrated areas (NCTM, 2000) but geometry has often been referred to the "forgotten strand" of mathematics (Lappan, 1999) and indications from the TIMSS study reinforce that very little geometry learning takes place from grade to grade (NCES, 2003). So what geometry do students, in general, and future teachers, in particular, need to know and what do teachers need to know to be effective in the mathematics classroom? Pierre and Dina van Hiele have suggested that students progress through levels of thought in geometry (van Hiele, 1986). Level one begins with students being able to visually recognize shapes; level two emphasizes the ability to recognize, analyze, and characterize shapes by their properties; level three involves informal deductions about classes of figures; and at level four, students should be able to establish theorems within an axiomatic system, (Clements, 2003). One can argue that mathematics instruction must offer the ability for students to learn geometry in ways to make the natural progression from one level to the next and that students cannot succeed at level four when their thinking is at level one or two. Ill-prepared teachers may reduce geometric content to rote memorization resulting in students who do not progress to higher levels of thought and will certainly not achieve geometric understanding (Clements, 2003).

While content knowledge is important, beliefs held by teachers about teaching and learning mathematics is equally, if not more a determinant of teacher classroom behaviors than content knowledge (Liljedahl, Rolka, & Rosken, 2007). Beliefs are powerful predictors of behavior, are resistant to change, and are sometimes not explicitly known to the belief holder (Mewborn & Cross, 2007). According to Mewborn and Cross (2007), teacher beliefs about mathematics that are not aligned with Principles and Standards (2000), and are not “healthy” for students, that is they are not conducive to learning. Commonly held beliefs which are not aligned with the standards include: believing that mathematics is computational; centered on getting the one, correct answer, quickly; and in the classroom, students are passive while the teacher is active (Mewborn, 2007). Beliefs which are aligned with the standards are described as: believing mathematics is problem solving, sometimes requiring significant amounts of time; the goal is making sense of the problem, process, and answer; and both teacher and students are active participants in the problem solving process. One prominent method for changing teacher beliefs is to immerse teachers in a constructivist environment; allowing teachers to experience what their students experience and modeling for teachers, methods which tend to produce the desired effects (Liljedahl, Rolka, &Rosken, 2007).

Many mathematics educators believe there are effective means to move teachers from what they know to what teachers need to know and be able to do in their mathematics classroom to maximize student learning. Furthermore, these means can help teachers align their practices and beliefs more closely with the recommendations outlined in the Principles and Standards (NCTM, 2000). With this goal in mind, an attempt to evaluate mathematics content courses at one university is in progress.

Methodology

At the university of this study, there are currently four mathematics courses for prospective pre-kindergarten through middle grades teachers: number and operation, geometry, algebra, and probability and statistics. In that geometry has often been described as needing attention at all school levels, the geometry classes were targeted for the first analysis of whether or not pre-service teachers were learning what we thought we were teaching.

Geometry content pre- and post-tests were administered beginning with the summer semester of 2009 with administration to one class and to two additional sections during the fall semester of 2009 , for a total matched pair sample size of 88. During the summer and fall semesters of 2009, all of the geometry content courses were taught by the same assistant professor. All pre-tests were administered on the first day of classes before any type of instruction had begun. All post-test were administered on the last day of classes in each semester.

Geometry content standards were considered from five sources. Documents which give recommended content topic areas for the initial preparation of teachers came from both the CBMS report (2000) and the National Council for Accreditation of Teacher Education [NCATE]/NCTM program standards (2009). Second, both the Mathematical Proficiency for All Students [RAND] report (2003) and the Principles and Standards for School Mathematics (NCTM, 2000) were considered for the big ideas in relationship to the types of geometric knowledge in which students should be proficient. Last, the Georgia Performance Standards provided more specific objectives to be covered in K-8 mathematics instruction. The basis of the first pre- and post test administered during the summer and fall semesters of 2009 was to see if students could recognize shapes, list properties of shapes, list common properties of shapes such as squares and rhombi, prove that the sum of the measures of the angles of a triangle is 180°, and consider alternative definitions for shapes. Most of the items required students to give short answers with the exception of two questions which required a proof or justification. A revision of the original pre-test was done before the administration in the spring of 2010. Some items were dropped because there was commonality in the types of information the students were asked to provide while others may have been rewritten in multiple choice format. For example, one of two questions was dropped which asked the students to write all the properties they felt were true for either a rhombus or a concave octagon. Other multiple choice questions, measurement topics in particular, were included such as "A square yard is equal to which of the following" and the answer choices included common mistakes that students had been observed making in the past.

A nine item, Likert scale of attitude towards teaching and learning was added in the assessment for 2010 semester. This scale was scored according to descriptions of classroom where mathematics learning is encouraged for process rather than content acquisition only as described in the Principles and Standards (2000). So the higher the score, the more closely the attitude or thoughts about teaching and learning mathematics was aligned with the Principles and Standards. An example of a typical question would be: "mathematics is about memorizing formulas and procedures." The respondent could select from strongly disagree to strongly agree. The following table lists the ideas that were associated with the attitude assessment (Mewborn & Cross, 2007; NCTM, 1989).

Beliefs most closely aligned with PSSM vision	**Beliefs not aligned with PSSM vision**
Goal is about understanding the process, the problem and the solutions	Goal is getting the "correct" answer
Teacher is the facilitator in the learning process	Teachers do problems, students mimic method: teachers are the only active ones in process
Mathematics problems can be different types: some solved quickly and others not	Problems should be able to be solved quickly
	As a field, mathematics is old and static (done by middle aged men, working alone)
Mathematics is about problem solving	Mathematics is a computational tool only
There might be multiple approaches to take with a problem	Teachers know the one, correct procedure to solve a problem
Most anyone can do mathematics or be a mathematician	Only certain individuals are good in math (probably genetic)

Results

A total of 187 pre-service teachers have been assessed over the last year in their geometry class for elementary and middle school teachers. The maximum geometry content score for this assessment was 49. The teacher pre-test scores ranged from 6 to 33 with the mean score being 23.7 and standard deviation of 5.3. The mean content score for the post-test was 29, standard deviation of 5.3, and scores ranging from 15 to 38.5. Using a paired sample t-test, the reported t-value was -9.7 ($p<.001$). Whereas statistically significant improvement in content knowledge was noted for most of the content questions, we were not convinced that teachers made progress beyond becoming better at the geometry procedures and formulas. For the two questions which required justification or proof, only one student out of 187 pre- or post- indicated some understanding of how to proceed towards a justification. Questions which required respondents to have a clear picture of the classification of polygons did not show significant improvement. An example of this type of question would be to name the properties of rhombi which are not true for squares.

During the spring of 2010, the pre-test was administered to three sections of geometry for teachers (N=91) with two different assistant professors teaching the classes. Pre-test scores ranged from 12 to 31, with a mean of 21 (N=91). The post- test for these students is planned for the end of the current semester.

The first attitude assessment was included on the assessment in 2010, with a maximum score of 36, indicating beliefs most closely aligned with the Principles and Standards vision for school mathematics classrooms. This survey will be administered at the conclusion of 2010 spring semester for assessment as to change in attitude over the course of the class.

Conclusion

With a statistically significant difference in pre and post content tests, geometry learning appears to have occurred for the paired sample group of students. Whether or not this is long term learning remains to be determined. Additionally, whereas the assessment was voluntary, the scores were somewhat disappointing overall. In particular only one student, out of 187, indicated any ability or willingness to correctly attempt a geometrical proof. On the posttest, less than 15 percent of respondents were able to acknowledge that all the properties of rhombi also hold true for squares or that all properties of acute triangles are true for equilateral triangles. It could be concluded the maximum van Hiele level for any of these pre-service teachers in this study was level 3 and most students were at level 2 at the end of the course. Level 3 is the abstractional/relational level of thinking which requires an individual to distinguish between necessary and sufficient conditions for a definition and requires informal arguments to justify conclusions (Clements, 2003). The question remains: is the level of knowledge for these pre-service teachers sufficient for them to be able to facilitate learning for their students beyond basic levels of understanding?

As noted earlier, the attitude survey was added in the spring semester of 2010 and those with higher scores would have beliefs more closely aligned with the vision of teaching and learning mathematics as described in the Principles and Standards for School Mathematics (NCTM, 1989, 2000). The resulting mean score of 21 was higher than expected for the initial assessment, based on student comments and conversations. This mathematics class, geometry, should be at least the third mathematics for teachers' course taken by these students. Additionally, many had taken at least one mathematic methods course where they should have been exposed to the practices recommended by Principles and Standards for School Mathematics (NCTM, 2000). Most respondents strongly agreed with the statement: students should be active in the problem solving process. On the other extreme, most students also agreed that the goal of mathematics should be getting the correct answer. This might indicate that pre-service teachers are hearing descriptors of a classroom environment conducive to mathematics learning, but will not or cannot relinquish the model under which they were taught. Future steps are to assess attitudes at the beginning of the mathematics content classes taught to pre-service teachers and track changes in attitude over the progression of mathematics content classes.

An additional benefit of this study has been the necessity of instructors to spend more time collaborating on content and activities. What activities and tasks move students further up van Hiele levels of understanding geometry? From the list of desired objectives for geometry content, what outcome objectives are not met and what geometric concepts have not been learned? What activities might accomplish the learning goals better than the activities currently being used?

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TRAVELING REPRESENTATIONS IN A FIFTH GRADE CLASSROOM: AN EXPLORATION OF ALGEBRAIC REASONING

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In this three-day teaching experiment along with follow up interviews, algebraic concepts related to pattern-finding tasks were examined with 25 fifth grade students. The specific focus centered on representations from a realistic mathematics education perspective, meaning a model "of" a situation toward a model "for" a situation. Within this context, certain situational models were found that seemed to travel and permeate throughout the entire class. Students were able to generalize and justify based on the models developed during whole class discussions. Several weeks after the teaching experiment, follow up interviews indicated that the representations generated were still prevalent in students' descriptions of the activities. Findings, analysis of findings, and implications of the study will be discussed.

Objective

The early algebra movement in the United States is gaining momentum throughout all areas of the elementary curriculum (Kaput, Carraher, & Blanton, 2008; NCTM, 2000). Recent research focuses on the need for preservice teachers to develop a better understanding of their own early algebra concepts (Richardson, Berenson, & Staley, 2009). Representations often play a vital role in helping students to reason algebraically (Presmeg, 2005; Smith, 2008; NCTM, 2000). The purpose of this report is to take recent research (Richardson, Berenson, & Staley, 2009) and extend it further through the eyes of fifth grade students using similar tasks. The research here is focused on fifth grade representations, namely the models generated within them, and the effects of a teaching experiment designed to make notable improvements in their algebraic reasoning (Lesh & Kelly, 2000).

Theoretical Framework

For 30 plus years, the Dutch Realistic Mathematics Education (RME) movement has provided a framework for a host of studies. The term *realistic* refers to problems having a context of 'real-world' or simply imagined (Presmeg, 2003). Centered on the idea of mathematics as human activity, Freudenthal (1977) insisted that context must play an important role in the teaching and learning of mathematics. Progressive formalization or mathematization is a key process within the RME philosophy and is comprised of students exploring mathematical ideas informally and then making gradual progress to more formal, higher level thinking. A variety of mathematical ideas are defined within progressive formalization and here the focus is on models. Models, in the context of this study, are defined as representations of problem situations that contain a realistic or imaginable context, possess flexibility, and can be re-invented by students on their own (Van den Heuvel-Panhuizen, 2003). As discussed by Presmeg (2003), Van den Heuvel-Panhuizen (2003) notes form-function shifts in the types of models students generate during their mathematical activity. Van den Heuvel-Panhuizen also draws from Streefland's (1985) work where he described a model *of* a situation to a model *for* a situation. Meaning, a student uses a model to investigate a particular problem but then later transforms the model to relate to other situations and/or to provide a way to better understand the situation at hand.

The work described here offers an adaptation of Van den Hevel-Panhuizen's and Streefland's research. For this study, *Context* accounts for how the student initially engaged in the problem, both through verbalizing ideas and modeling those ideas. *Flexibility* refers to how the student took the context of the problem and started finding patterns, hence working flexibly within the context. *Reinvention* indicates how the student re-conceptualized the problem. Flexibility and reinvention are closely related and difficult to separate. A unique contribution from the authors of this study is the idea of *traveling*, which means to capture the permeation of an idea within a class. A teaching experiment (Lesh & Kelly, 2000) is utilized along with task-based interviews (Goldin, 2000) to explore traveling and other moments of student investigations. Thus, the purpose of the research is to focus on specific representations that fifth graders used within a teaching experiment to solve algebraic pattern-finding tasks. Our specific research question is, how do representations of early algebra ideas travel over space and time in a fifth grade classroom?

Methodology

Design and Subjects

This teaching experiment focused on a fifth grade class at a primarily white, rural, science/mathematics focused elementary school in the southeastern part of the United States. A whole class teaching experiment was used because the researchers felt it was the most well suited setting to get at children's mathematical thinking (Lesh & Kelly, 2000). The researchers used task-based interviews because it was agreed that they were the most powerful way to focus on the individual student (Goldin, 2000).

In terms of mathematical ability, the class being observed had 25 average to above average students. During three consecutive days of instruction, video cameras focused on three student dyads recommended by the classroom teacher. Audio data were collected for all 12 dyads. Six weeks after the original three-day instructional period (about one and a half to two hours) the researchers returned to the school and did follow up interviews with students. In these interviews the researchers then asked them to extend their understanding of the original tasks.

In a larger study (Richardson, 2010), the focus is on 25 students but for this preliminary study, the focus is on both Dan's work and the work of other students related to his work. Part of Dan's work was whole-class and small group and another part was from his work from the one-hour follow up interview. Dan was confident and wrote clear explanations on his paper. He also stood out in the group because he was able to generalize rules, even on the first day of the teaching experiment.

Task and Instruction

The first task was called square tables (see Figure 1 for an abbreviated version). In this task students were asked to determine how many people could sit around a square table, if one person could sit on each side. They then determined how many people could sit around two contiguous square tables. The primary objective was for the students to be able to answer how many people could sit around *n t*ables, where *n* was an arbitrarily large number. The same question was asked on the second day (see Figure 2) but used triangles and the third day involved hexagons. Students were asked each day to organize their data in a table, using pattern blocks to build the larger models as needed, and to continue to describe any patterns they discovered during the teaching experiment. While this set of train tasks usually begins with triangle pattern blocks, earlier results of these perimeter tasks led us to change the hypothetical trajectory of the experiment to include the square tables in the first week, and then the triangle tables in the second week (Berenson, Wilson, P.H., Mojica, G., Lambertus, A., & Smith, R., 2007). The triangle task is also difficult for students to re-invent and give context to since one does not generally sit at a triangle shaped table.

During the follow up interviews the tasks were similar to the ones described above except that students were asked to consider tables that had two people sitting on each side instead of one. If they showed an ability to quickly grasp and generalize the new problem, they were then asked to examine tables with two people on each side of a table shaped like a pentagon, a shape that had not been part of the original set of three tasks. The researchers were interested in what insights they might construct during this new pattern finding activity.

Day 1 - If you have one square table, how many chairs will fit around the table if you have one chair on each side of the square? Two square tables? Three square tables? Do you see a pattern yet? If yes, write down a description of your number pattern.

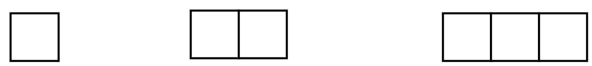


Figure 1. First task investigated by fifth grade students in an algebraic reasoning teaching experiment.

Day 2 - If you have one triangular table, how many chairs will fit around the table if you have one chair on each side of the triangle? Two triangular tables? Three triangular tables? Do you see a pattern yet? If yes, write down a description of your number pattern.

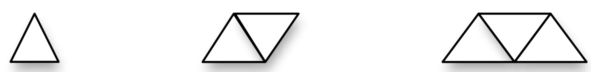


Figure 2. Second task investigated by fifth grade students in an algebraic reasoning teaching experiment.

Evidence and Analysis

Sources of data included video, audio, and written work of fifth grade students. The conversations were filmed using digital video cameras and conversations were also captured using digital audio recorders. The videotape interviews were transcribed and the transcripts were analyzed. Pseudonyms are used in all descriptions. Our overall analysis looks at modeling *of* the problem to modeling *for* understanding, which is demonstrated in Figure 3. It is what happens within modeling *for* understanding that is analyzed, so three areas of modeling were coded for: context, flexibility, and reinvention (Van den Hevel-Panhuizen, 2003). Table 1 lists our coding of Dan's work.

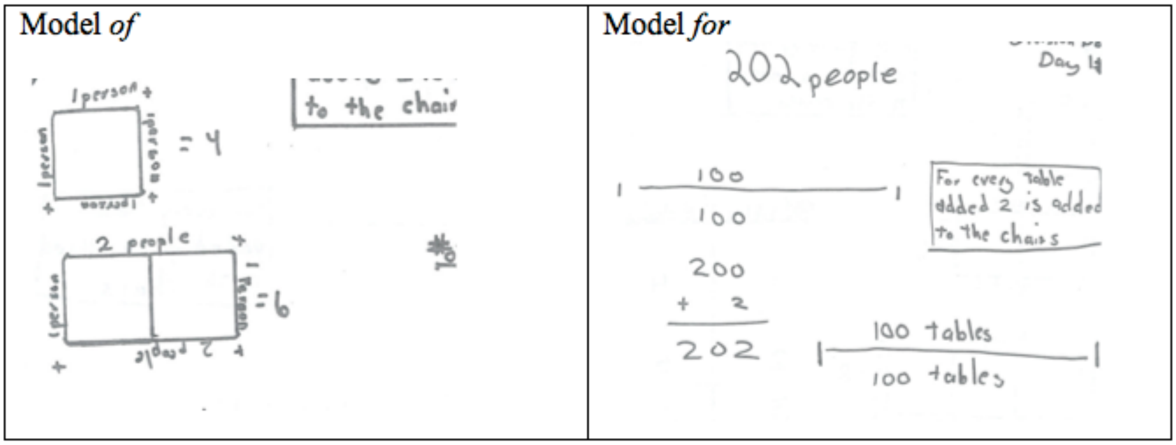


Figure 3. Dan's progression on the square tables task from 4, 6, and 100. Adapted from Streefland (1985).

Task	***Contextualizing the Problem***	***Demonstrating Flexibility***	***Reinventing Model***
Day 1 – Square Task	Drew squares and labeled them to represent the tables. Also made T chart.	Stopped drawing tables and wrote rule "For every table added 2 is added to the chairs."	Line drawn with 100 on bottom, 100 on top, and 1 on each end.
Day 2 – Triangle Task	Drew triangles and labeled them to represent the tables. Also made T chart.	Wrote rule, "every table added, one chair is added"	Did not draw reinvented model.
Day 3 – Hexagon, etc. Task	Drew no shapes. Simply filled in worksheet w/rules.	Wrote rules in written and symbolic form.	Did not draw reinvented model.
Follow-up Interview Task (pentagons)	Drew T chart, pentagons and labeled some parts.	Verbally expressed rules.	Line drawn with 200 on top, 100 on bottom, and 1 on each end.

Table 1. Dan's progression throughout the entire teaching experiment.

Results

There were two major results from the analysis of Dan's data. First, he reinvented the initial model of squares into a new model during day one and the follow up interview as a means to generalize algebraic patterns. Second, Dan's model had an impact on other student models in the classroom and six students utilized his model or a form of his model to describe their solutions to the algebraic patterning tasks. The results of each major finding are listed and the results detailed.

Dan's use of his reinvented model

Figures 3 and 4 demonstrate Dan's modeling during day 1 and during his final interview. Observations from the video data show Dan meticulously drawing the squares, labeling them, and then drawing a T table (not shown) next to his work. When asked how many people could sit around 100 tables, he started his work on a new sheet of paper, at which time his re-invented model was drawn. Other students kept filling in their T tables until they reached 100; others tried to find a pattern on their T tables, while some tried to multiply and add – all of which were notable occurrences for each student. However, Dan's re-invented model seemed to enable him to generalize a rule and justify that rule. For example, on day one, he wrote, "For every table added, 2 is added to the chairs." He went on to note 100 tables = 200 chairs + 2 = 202. Although on days two and three, he did not draw his reinvented model, he was easily able to express a generalization and justify his answers. On the follow up interview, though, he revisited his reinvented model and even used it to express his patterns for the pentagon task. This was surprising to the researchers because the students had difficulty in giving context to the other patterns that were not squares.

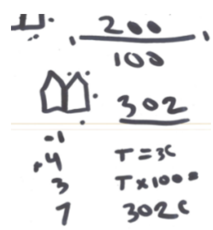
If you have 200 square tables, how many chairs will fit around the table if you have 2 chairs on each side of the square? 	If you have 300 pentagon tables, how many chairs will fit around the table if you have 1 chair on each side of the pentagon? 

Figure 4. Dan's reinvented model appearing again during the final interview questions.

The traveling of Dan's re-invented model

During day one of the teaching experiment, students were asked by the researchers to present their work to the entire class. Some students explained that the answer to the 100 table question was to find how many people could sit around 10 tables which was 22 and then multiply that by 10 to get 220. While pointing to his reinvented model, Dan presented his work stating that it was 100 x 2 + 2 = 202 but did a poor job in verbalizing why he had gotten this. Anna soon came up and re-drew his model in the same way and attempted to explain it but got stuck. It was at this point that the researcher asked, "Where does the plus two come from? Do you know where the plus two comes from in that drawing?" Anna was unable to elaborate more and looked to someone else to come up and continue. At which time Kevin came up and drew a form of Dan's model. It was a long rectangle, instead of just a line, with sloppy marks drawn inside of it to indicate the individual tables. He emulated Dan's labeling by writing a 100 across the top and a 100 across the bottom, with a 1 on each end. He stated, "These – there are 100 people on each side, but there's another person on each side – so there's two right here. So that's what it comes from."

It was after this explanation that students abandoned the 220 solution and researchers observed additional students drawing some form of Dan's model to illustrate the accurate 202 answer. In total, six students explicitly drew a form of Dan's model and wrote in words a generalization of the patterns they found. For example, Brenda wrote "Multiply tables by 2 and then add 2 to find the number of chairs." Another student, Kim, wrote, "100 people on each side of the tables so 100 + 100 = 200 then count the people on the end = 202." Two other students used Dan's model to verbally express their generalization but did not explicitly draw it. For example, Melanie wrote, "2 sides on a table (long) then multiply 100 x 2 for the 2 sides then add 2 end sides." Like Melanie, Stephen wrote out his rule in words as opposed to drawing it explicitly. He wrote, "You would take 100 and x it by 2 and get 200 + 2 gives you the answer." Upon close look at the video data and written work, it is fairly clear here that both Melanie and Stephen are using a version of the model that Dan had first come up with to state their own rules in word form even though they were not necessarily drawing this model on their papers.

Researchers Perceptions

The researchers perceived that each day within the teaching experiment built on the previous day since the students generally were more detailed in their descriptions as each new task was introduced. The follow up interviews, which were conducted six weeks after the teaching experiment, surprised the researchers because the students were able to easily engage in the tasks posed and remembered how they had worked their initial problems.

Discussion

The analysis of Dan's modeling, algebraic generalizations created from his modeling, and the impact of his work on other fifth graders, indicate both individual and whole class growth of algebraic reasoning. The researchers are reminded that learning is dynamic and additional time is needed to find out more about fifth graders' development of early algebra concepts and how modeling enhances those concepts. An examination of the modeling categories informs researchers and instructors of how a student can take a common model (e.g., squares, triangles, etc.) and re-imagine the model in a way that enables the student to make key algebraic generalizations. The ability to contextualize the square tables task, meaning students could imagine people sitting at tables, greatly helped the students re-invent within the problem. On days two and three, where the tasks were less contextual, meaning the same questions posed utilized triangles and hexagons, students were easily able to make a generalization by simply drawing the shapes and recording the patterns in a T table thus generating a rule. Posing a realistic question first, in this case the square tables task, enabled the students to later think about less contextual problems with ease, as indicated by the data.

Spending day 1 with only the square tables task was an important decision made by the researchers. Although the students were actively engaged in the task, they were reserved in expressing their findings. Their eagerness to share and their energy levels were much higher on days two and three and their ability to work more efficiently and less recursively were evident. However, their attention to detail and determination to work through several issues that arose on day one were instrumental in their successes for days two and three.

Putting the analysis of this teaching experiment, and Dan's work in particular, within the framework of realistic mathematics gives researchers valuable insight into how students conduct mathematical investigations and how they construct models. Examining Dan's work and seeing how much impact it had upon the understanding of the rest of the class demonstrates this clearly. His ability to work flexibly and reinvent problems enabled him to come up with creative and unique solutions that in turn benefited his peers. This permeation of an idea within a class of students or group of people is what the researchers have defined as *traveling.* It was quite clear through the course of this teaching experiment that Dan's idea permeated the entire class and helped his peers solve the problem in a similar manner as he did. Further more the researchers saw that this idea had longevity in that some of his classmates used the same strategy six weeks later in the follow up interviews. It is for this reason that the researchers consider the notion of traveling, and how the idea that traveled came into being as an important concept.

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TEACHERS CONNECTING DOTS: THE PATH FROM LEARNING MATHEMATICS TO TEACHING MATHEMATICS

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In this paper we analyze the disconnect we have identified between the mathematics taught in Mathematics in Mathematics Education (MIME) classes designed to help middle grades teachers deepen their understanding of mathematics and the mathematics taught by middle grades teachers who have been participants in these classes. We address the question: "What mathematics is important for teachers to know at the middle grades and why is it important?" from the researchers' and the participant teachers' perspectives.

Background and related literature

Over the last several years faculty from the mathematics and mathematics education departments at our institution have been collaborating to develop and co-teach a set of mathematics content classes for middle grades teachers. Local school districts identified a need for many of their middle grades teachers assigned to teach mathematics classes to have more content coursework to satisfy No Child Left Behind mandates for highly qualified teachers and approached us to develop and offer courses that would meet their needs. In developing these courses we recognized the following:

- many of these teachers were originally elementary licensed with minimal undergraduate mathematics coursework (86.6% in Ohio)
- the mathematics content needs of these teachers is different from the mathematics content needs of non-teacher mathematics students.

The team thus decided that something distinctly different was needed. To recognize this we developed the title: Mathematics in mathematics education – MIME. Initially the following set of courses were developed:

- MIME 60120 – Introduction to MIME
- MIME 60130 – Geometry & Measurement
- MIME 60140 – Patterns and Counting
- MIME 60150 – Foundations of Number Theory
- MIME 60160 – Algebraic Thinking and Mathematical Representation
- MIME 60170 – Probability and Statistics

It was decided that these courses would be co-taught by a mathematician and a mathematics educator, a decision supported by department chairs and university administration. This was seen as essential to recognizing the unique learning needs of middle grades mathematics teachers as supported by the literature in the field.

The Mathematical Association of America explains in its landmark document, *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences, 2001), that the mathematical knowledge needed for teaching is quite different from that required by persons in other mathematics-related professions. Teachers need an especially profound understanding of the concepts of mathematics so that they can teach it as a coherent, sense-making, reasoned activity. Ball and Hill (2009) also describe the knowledge necessary for teaching mathematics that is different from what is required by mathematics majors. This knowledge helps teachers make the critical decisions during a class to facilitate their students making sense of mathematics. This is, however, the view of the research community. In this paper we identify a disconnect between this view of the mathematics content needs of teachers and those of our students – the middle grades teachers who have participated in the MIME classes. We will explore the tensions involved as the three groups of participants in the class, mathematician, mathematics educator, and middle grades teachers confront the critical decision: What mathematics is important for middle grades mathematics teachers to know?

Methodology

We used a qualitative approach in this study. Each week the mathematician and mathematics educator assigned to the class met to design tasks for the purpose of helping teachers gain a broader view of the mathematics they are teaching. Extensive debriefing occurred during this time as instructors used the various artifacts from class – student class presentations, class discussions of mathematical ideas, homework assignments, weekly reflective journaling – to identify students' learning needs (Teaching Experiment Design, Cobb & Steffe, 1983). During the final week of class students participated in a one-on-one interview with a mathematics educator who was not assigned to teach the class. The following questions were used:

- What is Mathematics?
- What is your attitude toward mathematics?
- Do you believe that it is important for a teacher to understand more mathematics than they teach?
- Solve ____ and explain your thinking (here the student was presented with a problem related to the content of the class, provided by the instructors).

In this paper, along with analyzing a sample of teachers' responses to the interview questions, we analyze classroom events where the instructors, directly or indirectly, confronted this tension or disconnect between what they believed, based on prior research, were the needs of the students, and the beliefs of the middle grades teachers about their needs.

Results

In this section we share some examples of the data that we believe characterize the differences between the teachers' view of the mathematics they need to be successful teachers and that of the university group. The first data we will share come from the interview questions where we were trying to understand how the middle grades teachers perceive mathematics, their job of helping their students learn mathematics, and the reason for them to learn mathematics at a deeper level. In the second example we take an episode from a geometry class. This example illustrates both the level of mathematical knowledge and how these teachers perceive the depth of mathematics necessary to teach their students. Finally we present data from when teachers in another class were asked to analyze a student's solution to a division of fractions problem. We wanted to see if teachers would recognize the need to understand why such algorithms worked in guiding the learning of their students. What we present here is just a small sample of responses.

Data set 1: Interview questions: We present a summary of responses from three teachers to each question below.

Describe what you think mathematics is and give an example?

#1…A way of thinking about numbers. An example would be maybe number families like I am just giving you a basic example you know like the inverse of addition is subtraction and the inverse of multiplication is division, number families those kind of things.

#2…a process of numbers or equations. Finding maybe a function or a rule, patterns, many things come to my mind. An example would be the equation, how to solve the equation, the steps of being able to solve the equation to find the answer. Mathematics can be finding the rule. It is more to me totally numbers but it isn't numbers and that there is always a right answer and there isn't always one way to do it. There is many ways to do it and there can be different answers, nothing is concrete or for sure that there is no other answer because you can keep looking into it, there may be another answer in a different way.

#3… a method by which numbers are manipulated and concepts are manipulated to solve problems. They can be problems in business or daily life. There is a whole variety of aspects of mathematics. It is a method of communication between people who work in different fields. It is a standard way to solve problems; its data statistics, probability algebra. They all involve numbers and numeration.

How do you know when one of your students "understands" a mathematical idea? Elaborate or give an example.

#1…I might be giving you too simple of an answer here but the first thing I can think of is, if they understand it they can do it and then they can show me in another way it is correct. For example, if they did a multiplication or let's say a division problem that had a remainder, if they can flip that around and turn that into a multiplication problem and adding the remainder then I think they understand the concept.

#2…Because you always have the one student that asks you why or why does it have to be that answer? Why do you have to do it like that? Why does that function work, why does that rule work? Before it was really hard for me to answer that [these type of questions], I talked about this with the other math teacher I teach with and we said kind of the same thing, you were kind of like whoa! Wait! And you get those students and now it's like knowing more in depth of why it works and that you can prove it, the theory or function we can answer those questions now. There is always that one student that kind of throws you off and you want to say because I said so, you know you think that in your head but I would never say that, but now I think I will be able to approach it differently and keep learning. I would love to have more classes like this.

#3**…**OH…that's awesome. They all have different ways of demonstrating that. Sometimes they say "Oh, I get it now." I'll ask them to show me or explain it, to elaborate on it and to justify it. I know they understand it if they can give me a logical explanation for it. When they can come back to it later and use it, to apply it to something different, they really understand it.

Is it important for a mathematics teacher to know and understand more mathematics then they teach? Explain why or why not.

#1…Yes I do think it is important. One reason is because I think it gives the teacher more confidence. Another reason is for regular teachers, teachers of regular students, they could get students who are going to ask them explanations of things or they may have questions about something that is beyond really what you are teaching in the classroom and it would help to be able to explain that or work with them on finding an answer of what they are asking. Plus a lot of times you have to describe things in more than one way.
#2… Do want me to give an example of what I felt was a teachable moment, is that what you are saying? [Yes, that would be fine.] Well there again my Pythagoras, I'll focus here, that was first thing we started with in this class and we worked with angles and everything. One day after school after dismissal on my desk a student had taken the time and he has a lot of troubles in math anyways, he had taken a piece of graph paper and drew like a lake with a sail boat and it was me in the boat. He made a speech bubble and said "I hope I pass my math class." He put Mrs. G on the sea of Pythagoras, so to me I think he might not of understood everything but it is in his memory and maybe later on in life that will be one of his "brain bits." [How about in class, how do you figure out whether the kids, while you are teaching their mathematics, how do figure out whether they are really understanding or not?] I do a lot of hands on. First I lead a discussion and use the overhead then I do groups and do hands on and maybe each group I would give like a little project or problem to solve and then they discuss it and that kind of gives me an understanding with each group how they solved it or they had difficulties.
#3…[Chuckle] I'm a prime example of that. A lot of what we were learning is stuff I'll never teach on a 6th grade level, but it has helped me develop my understanding of some higher level concepts and higher level thinking. So if a question comes up from a student I might be able to explain that …this is why and how it works not just that you do it this way. This is the logic behind it. So I think that we need to understand ourselves what the reasoning is behind calculating or computing something in a certain way. That's a yes to this question!

In summary from this first set of data we believe that these teachers realize and verbalize that mathematics is more than just following rules; that they want their students to be able to solve problems and explain their answers. Further they believe it is important to know more mathematics than they teach, but the way they describe why they need to know more mathematics illustrates one disconnect between these teachers and the instructors in this project. They believe that it is important to know more mathematics so they can get the right answers, show their students how to do the problems, and explain things in more than one way. In addition, in the last response above, one teacher states, "I'm learning stuff I'll never teach at the 6th grade level" which hints that she isn't exactly sure why she needs to understand the mathematics she is teaching at a deeper level. The MIME instructors believe it is important for the teachers to know the mathematics they teach at a deeper level so they can challenge their students' thinking about mathematical ideas to help them to become mathematical thinkers and thus determine whether their own answers are correct and not have the teacher verify their solutions. This is one of the differences we have noticed in the data, suggesting that the difference is fundamental.

Data set 2: Classroom episode

The second piece comes from an exploration task used in the geometry class using the software program *Shape Makers* (Batttista, 2002). In working with the quadrilaterals unit the middle grades teachers expressed frustration that different texts defined a trapezoid differently, some using the definition, only two sides parallel, while others not specifying the "only" but instead, saying "at least." Their approach to solving this problem was to "fix" the textbooks so that all agreed. The instructors saw this, not as a problem to be "fixed" but as an opportunity for discussion about the nature of what it means to do mathematics. The mathematician tried to use this as a way to explore the idea that definitions are not always agreed upon and fixed, but are modified depending on the topic, or that the mathematics community cannot always agree on how something should be defined. The mathematics educator tried to have the class see how the class discussion of the issue had led to a rich exploration of the properties, not just of trapezoids, but of all quadrilaterals. Many of the teachers however, remained frustrated, saying that this would be "too confusing" for their students, that the definition needed to be clear and unambiguous. In the words of one student as she reflected on the discussion in her journal later:

> This works well this year, but if our class agrees on the first definition, won't it cause confusion in future years if the class agrees on the other definition? While we as adults can reason through these definitions and make sense of them, some students would have a very difficult time with this. I know I can't change textbooks, but I can <u>at least try to reduce the confusion</u> as much as possible. I <u>always thought of math as a very exact, measurable, provable science,</u> but it seems like <u>this whole definition "thing" this is contrary</u> to this idea. (authors' underline for emphasis)

In summary of data set two we would like to point out the underlined phrases in the quote from the teacher's journal entry. We believe this again illustrates the teachers' perspective of their job is to "reduce confusion" and provide clear explanations of mathematics for their students. These teachers didn't see that engaging in thinking mathematically about definitions was productive for them or that it would be beneficial for their students' mathematical development. This is somewhat contradictory to their explanations given in question 1 of data set one.

Data set 3: Final interview question: Solve ____ and explain your thinking (here the student was presented with a problem related to the content of the class, provided by the instructors).

Here we share a response to the last interview question where the middle grades teachers were to analyze a student's (Patrick) approach to finding the solution.

Find the answer to $3/5 \div 2/3 =$

> Pat says "you get common denominators like this: 3/5 = 9/15 and 2/3 = 10/15,
> Now you look at 9/15 ÷ 10/15 that is 9/10 ÷ 15/15. This is 9/10÷ 1. Any number divided by 1 is the number. So your answer is 9/10."

Teacher responses: [*interviewer comments*]
How do they go from 10/15 to 15/15? [*Interviewer explains the part after that is in the procedure above. Pause again…She then did the problem using the standard procedure and said that*] I did the problem the way I do these problems and I do get the same answer. I was just trying to figure out his reasoning behind this. [*Interviewer asks how Pat's process connects with the process she used to solve the problem*]. I'm still confused how he gets 9 divided by 10 over here [referring to the part following that is in Pat's procedure]. I'm not understanding how we get to this. [*after some clarification the interviewer asks again if this procedure will always work*]. Yes, I would say that it is a correct procedure. [*Does it work all the time?*] Well I would try another problem. [*she picked 2/3 ÷ 3/4*]. I would get common denominators 8/12 ÷ 9/12. Which you have to do first before you divide. Then you have 8/9 ÷ 12/12 which is 8/9. [*She went on to do the standard procedure to verify that it worked.*] I would assume that yes it would work. [*Interviewer asks again how she knows this works for all fractions*]. My understanding of fractions is that the property of fractions …is all the same. If it works…I feel that I need to develop a relationship somewhere between here the way I'm used to solving and the way he did it. And if I develop that …if I would sit here and figure it out which is what I was trying to do at first…whatever that relationship is I would assume that would be true for all division of fractions. [*Interviewer...reviews Pat's procedure and she writes the initial problem vertically*] And you can use reciprocals because if you take the reciprocal of the denominator this becomes one…which is essentially what you are doing. [*Interviewer reiterates what she has just done. She goes on to explain that*] if you multiply by the reciprocal of the denominator these cancel out and you get one. Wait…wait…so I would assume that it would work for all fractions. I don't have a theory or formula to back me up but I'm saying yes. I would need some more time to prove that this was true.

We see in this example that the middle grades teacher takes the stance that Patrick's method works because it gives the same (correct) answer that her own procedure would give. This is a different type of reasoning than thinking about the logic of the relationships that Patrick used in his method. We would suggest that it is actually a somewhat simplistic form of reasoning that is frequently used by middle grades students to justify why different procedures work. Patrick himself may not have a reason beyond the level of "it works!" We would claim that the middle grades teacher has a distinct responsibility to provide an opportunity for the class to explore fraction concepts in order to figure out what Patrick is doing, how he is thinking, and justify the viability of the procedure from its inherent mathematical logic. This is one aspect of what we believe it means to know mathematics at a deeper level in order to teach middle grades students, or students at any level. There are a number of other examples from class discussions with these teachers, where instructors try to press them to developing mathematical explanations and perhaps “proofs” for why procedures work, to be confronted with the argument from the teachers: “This would be too difficult for my students to understand.”

Conclusions

The differences between the instructors meaning of understanding the mathematics that is involved in teaching middle school mathematics and that of the middle grades teachers themselves is significant. These differences can be characterized by knowing how to do things as opposed to knowing how and why things in mathematics work and are connected. The mathematics needed by teachers as perceived by teachers at the middle grades has to do with their background in mathematics and their view of the day-to-day requirements placed on them. The commonly held belief that mathematics is a set of rules to be learned and used for prescribed problems has led teachers we interviewed to adopt a “my grade plus one” attitude regarding what mathematics is needed for them to be successful mathematics teachers in the middle grades. For example 7^{th} grade teachers responded that they indeed needed to know more mathematics than what they taught at that grade level, but went on to say they just needed to know 8^{th} grade mathematics so they could get the students “prepped” for the next class they are to take.

This perception of mathematics and their beliefs that mathematics is for the most part procedural creates their personal views of what mathematics is necessary to teach in the middle grades. This presents problems when planning to teach content courses for middle grades teachers. Doing the mathematics for the instructors means thinking mathematically; which is opposed to “you do it that way because it always works.” Engaging students in meaningful mathematics for the instructors is a way to help students become better at using the mathematics they know to help them figure out what they don’t know; for these middle grades teachers it provides more ways of getting the answer. We conjecture that teachers believe more ways of solving the problem, using manipulatives, and using “real life examples” is sufficient for their students to learn mathematics. We also conjecture that teachers do what they believe will best help their students be successful in mathematics.

How do you plan to meet these teachers with tasks you believe challenge their thinking about particular mathematics ideas and at the same time help them to see this mathematics is necessary for them to be effective in teaching middle grades mathematics? The language of mathematics used to communicate mathematical ideas is a major goal of our courses and one reason we cannot simply tell teachers what knowing mathematics at a deeper level means. Our attempts at introducing problems of this type have usually brought up complaints (mild revolts as described by our co-teaching mathematicians) from teachers that “these problems are too hard for our students.” While on the one hand the teachers say it is important to involve students in activities, real world problems, and to have them justify their answers, the evidence in these examples from our data illustrate a complex set of conflicting tensions. Teachers believe knowing “my grade plus one” and their statements that learning more mathematics helps them provide more answers or be better able to explain the mathematics to their students is different from what research indicates the depth of knowledge in mathematics provides teachers. What may be most troubling is that these teachers seem to be using words consistent with a constructivist perspective of teaching and learning mathematics, but hold beliefs more consistent with a behaviorist.

Doing the mathematics for the instructors means thinking mathematically. As teachers they provide their students with rich problems and then provoke their thinking with questions they know will eventually help them to make sense of the ideas. We suggest it is “unhealthy” to provide students with “hands on activities” and allow them to learn multiple ways of solving problems unless they are guided with productive questions that will help them decide which procedure is more efficient or whether their own answers are correct (McClain & Cobb, 2001). So we are struggling to negotiate with the middle grades teachers what mathematics is really important for teachers to understand and what this understanding entails.

Research-based curricula assume that teachers understand the connections between and among mathematical ideas. It is important that these people know what questions to ask and when to call to students’ attention these important connections; without this type of understanding teaching cannot do justice to this type of curriculum. There is no exact map for teachers to follow that can help an individual deal with any or all of the situations they can get into with any curriculum. There is another more personal part of teaching which combines the mathematics, the pedagogy, and the timing, or knowing when to ask questions to stimulate learning that can’t be put into a recipe (Steffe, 1990). This way of combining the essential elements of teaching mathematics can be learned through practice, reflection and discussions, deepening one’s mathematical insight, and making changes to one’s teaching based on data from one’s own students thinking. Our dilemma: How do we, through our MIME classes, help teachers deepen their own mathematics in ways that confront these disconnects and meet their needs as teachers?

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CREATING A CONTEXT TO PROMOTE ALGEBRAIC REASONING

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This paper reports results of qualitative data analysis from a three-days teaching experiment that aimed at teaching algebraic reasoning to fifth graders (n=25). Data were collected through observations, students' artifacts and videos. It focuses on the teacher practices that created a context for successful algebraic reasoning. These practices included encouraging use of multiple representations and strategies, orchestrating sequences of students' responses to be presented to the whole class, and promoting meaningful explanations and justifications. Implications from the study findings are also discussed.

Literature Review

Algebraic reasoning may be described as functional thinking with an emphasis on the relationships between quantities and ways of representing such relationships (National Council of Teachers of Mathematics (NCTM), 2000). Several studies show that elementary school students are capable of algebraic reasoning (Lannin, Baker & Townsend, 2006; Warren & Cooper, 2007). However, Lannin et al. (2006) explain that students may have difficulties in making generalizations because of teaching approaches that place an emphasis on algorithms rather than conceptual orientations. For this reason, and because algebraic reasoning goes beyond simply manipulating algebraic symbols, teachers are challenged to transition from the traditional teacher-centered practices to teaching practices that make algebraic reasoning more accessible to students. NCTM (2000) confirmed this need by requiring teachers to create contexts that allow students to share and evaluate each other's ideas. However having reform based standards and resources for teaching does not guarantee teaching practices that promote algebraic reasoning (Depaepe, De Corte & Verschaffe, 2007). Similarly, engaging in algebraic tasks does not assure algebraic reasoning (Earnest & Balti, 2008). Hence there is a need to identify teaching practices that promote algebraic reasoning.

Mueller and Maher (2009) explain that some teacher actions are necessary to elicit reasoning that leads to successful justifications. These actions include creating contexts in which students explain and evaluate their reasoning, and also other students' reasoning. These practices are rooted in constructivist theories and classroom social norms that are associated with explaining, making sense of other classroom members' explanations, justifying, small group and whole class discussions, questioning alternatives and argumentation (Yackel & Cobb, 1996; Cobb, Stephan, McClain & Gravemeijer, 2001). Cobb et al. (2001) refer to these actions as sociomathematical norms when they support learning of mathematical concepts, and the norms may be initiated and supported by teacher actions (Cobb et al., 2001; Martin, McCrone, Bower, & Dindyal, 2001). A teacher's decisions and actions create sociomathematical norms that may promote or hinder students' conceptual understanding of mathematical concepts (Elliott, Kazemi, Lesseig, Mumme, Kelley-Petersen, & Carroll, 2009; Tatsi & Koleza, 2008). For example, as discussed earlier, norms with algorithmic orientations may hinder algebraic reasoning, while as other norms may create a context that promotes algebraic reasoning.

The importance of productive sociomathematical norms is well documented in literature. When students are given a chance to assess their solutions and explain them to their peers, their cognitive autonomy is positively influenced (Lo, Wheatley & Smith, 1994; Yackel & Cobb, 1996). Other studies have shown that sociomathematical norms of what counts as a different, sophisticated, efficient and acceptable mathematical solution and justifications are associated with increased conceptual understanding of mathematical concepts (Cobb, et al., 1991; McNeal & Simon, 2000; Simon & Schifter, 1993). These sociomathematical norms are also associated with developed problem solving skills and mathematical reasoning (White, 2003). With such value, it is important that teachers establish norms that promote mathematical understanding. Despite such importance, Depaepe et al. (2007) reported that one teacher out of the ten sixth grade teachers observed in their study of mathematics classroom cultures established a norm by communicating to their students that there are multiple ways of solving mathematical tasks.

Establishing sociomathematical norms that promote understanding may be a challenge to many teachers. When teachers establish the sociomathematical norms, they are faced with developing a balance between fostering discourse community and meeting the content demands (Sherin, 2002). The students may also experience cultural clash, where newly introduced norms bring confusion to their orientations and frustrate them in the mathematics lessons (McNeal & Simon, 2000). Moreover, creating a context in which students develop conceptual understanding does not simply result from students explaining the solutions, but from context that promote explanations and justifications that are meaningfully built from mathematical argument (Kazemi & Stipek, 2001), a concept Clark, Moore and Marilyn (2008) refer to as speaking with meaning.

In the face of challenges of implementing algebra standards at elementary school level and of implementing NCTM (2000) standards of creating communities that foster reasoning, this paper aims at contributing solutions to these challenges. The question of study here is to identify and define teacher practices that created a context that promoted algebraic reasoning in a fifth grade classroom.

Framework

In conducting the research and analyzing the data of this teaching experiment with fifth graders, an interpretive framework of Cobb et al. (2001) was used. This framework incorporates the theoretical ideas from both social and psychological perspectives to understand the mathematics classroom. Specifically, the social perspective looks at the shared mathematical thinking and reasoning through three constructs: (a) the classroom social norms, (b) sociomathematical norms, and (c) mathematical classroom practices (Cobb, 2001). The data analysis in this research focused on the sociomathematical norms.

Sociomathematical norms are those norms specific to a mathematics classroom (Yackel & Cobb, 1996; Dixon, 2009; Hershkowitz & Schwartz, 1999; Kazemi & Stipek, 2001; Levenson, 2009; McClain & Cobb, 2001). Yackel (2001) defines sociomathematical norms as classroom standards that guide meaningful and sophisticated mathematical thinking and understanding. Clark et al. (2008) also state that sociomathematical norms are normative behaviors found in the mathematics classroom that account for acceptable mathematical solutions and behaviors. Many researchers offer the following examples of sociomathematical norms: providing mathematical explanations and arguments that are conceptual versus procedural in nature, developing relationships between multiple strategies, explaining contradictions in errors or varied strategies, allowing opportunities to re-conceptualizing a problem, and formulating collective arguments within the classroom while respecting student's mathematical ideas (Clark et al., 2008; Hershkowitz & Schwartz, 1999; Yackel & Cobb, 1996; Kazemi & Stipek, 2001).

However, McClain and Cobb (2001) suggest that sociomathematical norms are based on the debates among teachers and students to determine which norms are relevant, complex or multifaceted, well-organized, and adequate or satisfactory. This implies that sociomathematical norms are not constant, but they change according to the particular classroom culture and the dispositions of the students. Because students learn as they interact with others by collaborating, explaining and justifying their reasoning in the classroom (Mueller, 2009; Martin et al., 2005; Fransisco & Martino, 2005), teachers must create opportunities for students to communicate their thinking and evaluate others ideas. Therefore, our interpretation of the teaching and learning of algebraic reasoning in the teaching experiment on fifth graders is informed by the sociomathematical norms that emerge from the data collected.

Methodology

This teaching experiment in a rural, fifth grade classroom was implemented on three consecutive days. The classroom teacher reported that the twenty-five students in the class were of average to above average ability. Eleven of the students were female and fourteen were males. A university mathematics educator was the instructor using a series of isomorphic pattern finding tasks that focused on algebraic reasoning. Students selected research names for the teaching experiment and these names were used throughout the experiment on all sources of data, including video, audio, transcripts, and artifacts produced by the students. The time allocated to the tasks was ninety minutes to two hours each day.

Students used pattern blocks, paper, and pencil to model the following task: If one person can sit on each side of at a square table, how many people can sit if we add another square table? Three square tables? One-hundred square tables? Can you generalize a rule that will work for any number of tables? How could you convince me that your rule works? A model of this task is shown in Figure 1.

Figure 1. Model given on Day 1 of the teaching experiment.

The task was modified on subsequent days using triangles, hexagons, and pentagons. Generally, most students were able to generalize an explicit rule ($p=2n+2$) and apply their rules (Richardson & McGalliard, 2010). A few students were able to symbolize the rule but most expressed their rules with words. By the second day of the teaching experiment most students were able to offer justifications of their rules, explaining why they multiplied and added two to the number of tables. These results were replicated in individual interviews six-weeks after the teaching experiment. Similar studies of algebraic reasoning with elementary students, as reported by Richardson, Berenson, and Staley (2009), have not always yielded these successful results.

Results

To better understand why the results of the teaching experiment were favorable in terms of students' algebraic reasoning, for this report, the context within which the algebraic reasoning occurred was studied. The sources of data were transcripts of the instructor's and students' utterances and videos of the class. Data were analyzed to look for common themes and four sociomathematical norms of the teaching experiment classroom were identified. Then transcripts were re-examined and coded to identify the different practices of the instructor that was associated with each of the four norms.

Encouraging multiple solutions and strategies.

In creating the context for algebraic reasoning, the teacher developed the notion that there is more than one strategy to the solution. This was evidenced in drawing the models to represent the context of the task, in naming variables, in expressing and justifying the rules. The episodes and explanations below show how the teacher facilitated these mathematical practices.

When building the models, the university teacher (UT) encouraged the students to choose any representation that made sense to them as long as it preserved the data as in this episode.

UT: *So, Mia has just done a sketch. I want to show you her work. This tells us the information that we need. So, she just drew them freehand. Alright, so you can see where she's put – next to her drawing she's put one table and four, two tables and six, three tables and eight, four tables and ten and then she has a little 1's around where the chairs would go and the people would sit, so that's one way of doing it… Wilson has done it another way. He's traced everything because you can see that they're all nice and uniform. And next to it he's written, one table four people, two tables is six. So, that's another way of doing it. And both of these are right.*

The teacher continued to support this mathematical practice by repeatedly saying "*there are multiple ways of naming the variables*." The students accepted this notion of open endedness and extended this norm to the ways in which they expressed their rules in multiple ways as shown in Figure 2.

Figure 2. Expressing generalizations in multiple ways.

Orchestrating the students' responses

When the students were working in pairs to find a rule, the university teacher walked around the classroom to note the work of each pair of students. During the first task she found that most students wrote recursive rules but found one pair of students that wrote an explicit rule. This information informed her choice of students who would present to the large group and in what order. In other words, she orchestrated the order by having the recursive generalization first and the explicit generalization last.

King: *I did my table* (shows his t-table to the class on the document camera) *and every time you add a table* (square) *you add two people because when you put a table and a tables, you can't like this – you can't put anybody right here, so you can only put two here and here.*

UT: *So you can only add two people, huh?*

King: *Usually you can add four, one, two, three, four,* (indicates his model of one square table) *but you can only have six there when they're like that* (indicates his model of two tables).

UT: *Okay. Good, alright. So we've got Brenda. Brenda, do you want to come up and share?*

Brenda: (Brenda puts her work on the document camera for the class to read) *I did if you add one table, you add two chairs because and then multiply the tables times two, so two times one is two. So then I did T, which is the tables, times 2 plus two equals chairs*.

UT: *Okay, say your description in words one more time. Where it says "rule." Say that one more time.*

Brenda: *Multiply the tables times two and then add two to find the number of chairs.*

UT: (Addressing the class) *Think about that one more time, does that work?*

Promoting the sociomathematical norm of justifying solutions.

When the students had contradictory answers (202 and 220) to the number of people who would sit around 100 tables, she did not tell the students the right answer, rather she used the contradiction as an opportunity for students to use mathematical arguments to decide for themselves the right answer.

UT: *Alright, we have 202 or 220. How are we going to be sure which is right?*

Students spoke with one another about the strategies they used to arrive at their answers. The teacher then asked the students to explain their thinking to the whole class.

UT: *This table thinks it's 220. Could you* (referring to those with 202 as a solution) *prove to them that you're right and they're wrong?*

After the students attempted to justify to each other, the teacher checked to see how many student remained unconvinced, positioning students to give more convincing argument.

UT: *How many are convinced that it's still 220? Alright, how many are convinced that it's 202? How many aren't sure?* (Students were raising their hands in response) *You're not sure, you're not sure. So, let me give you the next question that I'm going to ask, alright the next question is can you show me why your rule works for either 220 or 202? That's called justifying.*

The solution (202) was accepted when one student gave an argument that convinced each member of the classroom that the answer to 100 tables was 202 chairs. This practice was evident throughout the lesson.

Promoting sociomathematical norm of "speaking with meaning."

The students had multiple ways of justifying their rules and solutions. The teacher positioned them to justify the rule p=2n+2 conceptually by referring to the contexts and models.

Anna: *I did the same thing that Dan did, but I had a – there's a way to prove that 100 people are on this side of the table and on that side of the table because on this sheet there's the number of tables or the number of people at each table* (refers to the input/output table).

UT: *Okay, does anybody know where the plus two comes from? King, do you want to get up and show me?*

Kevin: *These – there are 100 people on each side (*refers to the sides of the train of squares)*, but there's another person on each side – so there's two right here. So that's what* (where) *it* (202) *comes from.*

Conclusion

The aim of this study was to identify teaching practices that created a context for algebraic reasoning. From the analysis, the teacher endorsed the use of multiple strategies, a practice that encouraged the students to reason and choose a strategy that was meaningful to them. In orchestrating student responses, the teacher chose the strategies that were less efficient in solving the problem to be discussed before the more efficient strategies. In that way, the students not only appreciated that the problem can be solved with multiple strategies, but it also gave them a chance to make sense of the reasoning behind the more efficient strategies and why they work. The acceptable solution in the class was one that had a justification that was convincing to all the students. The teacher used the contradictory answers in the class as an opportunity for students to justify their solutions to one another rather than have the "teacher" judge the answer. Further, she promoted student justifications that were conceptually oriented by asking questions that required the students to refer to their models.

Algebraic reasoning may be helped by creating contexts that support sense making and conceptual understanding of the generalizations. From this teaching experiment, it was learned that these discussed practices promoted algebraic reasoning. NCTM (2000) emphasizes teaching practices that promote reasoning and contexts in which students communicate and assess their mathematical ideas. For teachers to do this successfully, there is a need for future research to explore further the teaching practices that promote algebraic reasoning.

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THE ROLE AGE PLAYS IN PRESERVICE ELEMENTARY TEACHERS' BELIEFS ABOUT MATHEMATICS

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Using a qualitative approach to research, the researcher examined six traditional and six nontraditional (based on age) preservice elementary teachers' beliefs about mathematics. All participants were enrolled in one of three mathematics content based courses designed for preservice elementary teachers at a doctoral granting university in the western United States. Data collection consisted of three forms: bimonthly classroom observations, preservice participant interviews, and instructor interviews. Data analysis suggested that participants' opinions about mathematics could fit into one of three categories: standards based, nonstandards based, and a combination of standards and nonstandards based. In particular, nontraditional participant beliefs spanned the three classifications, while all traditional participant beliefs fit into the combination category. An additional finding suggests that preservice elementary teachers' views about mathematics may be influenced by mathematics content courses designed for preservice teachers, family, maturity, and public school experience.

Introduction

Current trends in mathematics education, including documents such as *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) stress the importance of conceptual teaching and learning in the classroom. These goals are vital for students' true understanding of mathematics topics but are often difficult to achieve, even with college education that focuses on standards based mathematics instruction. Research shows that teachers who learned conceptually in preservice teacher programs still may emphasize procedures for understanding in their classrooms (Eisenhart et al., 1993). Teacher education programs need to learn more about what preservice teachers believe about mathematics to be able to help teachers see the value in conceptual learning, not only in their college work but also in teaching students. Since the emphasis in mathematics education evolves over the years, these belief systems may be different for nontraditional, those aged 25 years old or older ([National Center for Education Statistics [NCES], n.d.), and traditional preservice teachers, which is part of the focus of this research.

A fundamental study to the development of this research was the work conducted by Raymond (1997) with inservice elementary teachers. In part of her work, Raymond investigated inservice teachers' belief structures about mathematics, mathematics learning, and mathematics teaching. She utilized tables of criteria to suggest that participants' views fit into a certain category that ranged from traditional (nonstandards based) to nontraditional (standards based) views. For this particular study, the researcher used Raymond's classifications for traditional beliefs about mathematics, which included descriptions of mathematics to be "an unrelated collection of facts, rules, and skills" and mathematics as "fixed, predictable, absolute, certain, and applicable" (p. 556). On the other end of the spectrum, Raymond described nontraditional mathematics as "dynamic, problem driven, and continually expanding," as well as the possibility of it being "surprising, relative doubtful, and aesthetic" (p. 557). Unlike Raymond, the researcher of this study referred to "traditional" mathematics as "nonstandards based" and "nontraditional" mathematics as "standards based," which are terms aligned with the views of NCTM (2000).

Therefore, using qualitative means, the researcher addresses the following research questions:

Q1 How do traditional and nontraditional preservice elementary teachers define mathematics in regards to standards and nonstandards based mathematics beliefs?

Q2 How do the opinions of traditional and nontraditional preservice elementary teachers change over a semester long preservice elementary teacher focused mathematics course?

Literature

Although the literature is sparse on studies about traditional and nontraditional preservice elementary teachers and their beliefs, there are several articles about adult students, elementary preservice teachers' views about mathematics, and inservice elementary teachers' views about mathematics. In the following paragraphs, some of these articles relating to these three areas are discussed.

Since this study focuses on the different viewpoints of traditional (younger) and nontraditional (older) preservice teachers, literature about adult students of various ages is an important genre to consider. Research shows that adult students are able to achieve at the collegiate level (Richardson, 1994) but may contend with certain burdens during their college career. These obstacles may be different from the typical younger learner problems, which may include commuting long distances and children (Schuetze & Slowey, 2002). Even though adult students may have hindrances in their way to success, they also possess motivational factors, such as obtaining better employment and support (Blair, et al., 1995), to encourage them to achieve at the collegiate level.

In addition to articles about adult students, research involving preservice or inservice teachers and their views about mathematics (Ball, 1988; Collopy 2003; Ma, 1999; Philippou & Christou, 1998; Raymond, 1997; Sztajn, 2003; Thompson, 1984) need investigating. In each of these studies, the researchers discovered participants' beliefs about mathematics that ranged from procedural to conceptual in nature. Various opinions about mathematics spanned the literature including the following: "There isn't a universal explanation" (Ball, 1988, p. 16); "Although Teresa (an inservice teacher participant) believes higher-order thinking skills are important, basic facts, drill, and practice are at the core of what she perceives as her students' needs" (Sztajn, 2003, p. 64); "Math is like a game...It's learning the patterns to it" (Collopy, 2003, p. 295); and, "Any statement or answer in mathematics was either right or wrong" (Philippou & Christou, 1998, p. 202).

Methodology

For the study, the researcher chose 12 female preservice (6 nontraditional and 6 traditional) elementary teachers who enrolled in one of three mathematics content courses (MATH 100, MATH 200, or MATH 300) designed for preservice elementary teachers at a doctoral granting university in the western United States. The researcher selected three specific classes, one from each level, to conduct the study. The researcher chose these three classes because of the larger number of possible older preservice teachers willing to participate, as well as the similar belief systems the three instructors of the courses held.

The three instructors of the classes under observation utilized mainly group work and conceptual teaching strategies rather than lecture as a means to teach the courses. Each believed in the value of conceptual and procedural learning, which was evident from class observations and instructor interviews. During a typical class, the instructor would begin the class with an overview of the material and allow a portion of the class time to be spent working on group activities. Although all three classes utilized manipulatives, MATH 100 participants used manipulatives the most, while MATH 300 used them the least. The mathematical focus for each course under study varied, where the instructor of MATH 100 focused on number sense, MATH 200 emphasized probability/data analysis/algebra, and MATH 300 stressed geometry.

From each of the three chosen classes, the researcher picked four participants, two traditional and two nontraditional. Even though the researcher could choose participants under 30 as part of the nontraditional participants, she wanted participants who were at least 30 to hopefully see more of a distinction between the two groups. The traditional participants' ages ranged from 18 to 20 with a mean age of 19 and the nontraditional participants' ages ranged from 31 to 53 with a mean age of 37.

For the data collection, the researcher interviewed each preservice teacher participant two times, once at the beginning and once at the end of the semester. Each interview lasted approximately 45 minutes and included various types of questions, such as ones regarding preservice teachers' philosophy about mathematics and questions from Raymond's (1997) research. Two sample interview questions about participants' beliefs about mathematics consisted of the following:

1. "What do you think mathematics is all about" (Raymond, 1997, p. 555)?
2. Describe the degree you feel mathematics is
    a. "dynamic/static,
    b. predictable/surprising" (Raymond, 1997, p. 561).

In addition, the researcher interviewed each instructor twice during the semester. These interviews occurred after each round of participant interviews and lasted approximately 30 minutes each. During instructor interviews, the researcher asked about various topics including preservice teacher behavior/progress in class, as well as class routines and preservice teacher comments made during their interviews. As an additional form of triangulation (Mertens, 2005), the researcher observed each instructor's class under study twice a month and documented on class activities, such as preservice teachers' struggles with mathematics and their interactions with other classmates.

After gathering and transcribing the data, the researcher used the qualitative software package *Nvivo* for coding purposes. She also created code names starting with an "N" for "nontraditional" and a "T" for traditional" for all the participants. Through the use of open coding (Corbin & Strauss, 2008), the researcher constructed a list of code words from the data. She added some new code words, but several came from the two pilot studies (Wheeler, 2009) from earlier semesters.

Findings

From data analysis, the researcher discovered three of the nontraditional participants held standards based beliefs about mathematics, while two nontraditional participants held nonstandards based opinions. All six traditional and one nontraditional participant believed mathematics to be a combination of standards and nonstandards based mathematics.

The three nontraditional participants who held standards based opinions about mathematics were enrolled in either MATH 100 or MATH 200. One of the two nontraditional MATH 100 participants who fit this classification was Nancy. Of all the preservice teachers in the study, Nancy, aged 53, visibly struggled with mathematics the most in class and commented about the difficulties of the class during interviews. Even though Nancy had trouble in class, she remained diligent in her efforts to work with the manipulatives and conceptual ideas the instructor presented. In one of her interviews, Nancy described how she felt mathematics was surprising and provided an example from a MATH 100 reading of the way people in India utilize different counting techniques to teach the number system.

> Well, one of the things our instructor handed out was really interesting to me. I think it was when you saw India thousands and thousands of years ago. It just kind of clicked in my head how the number system, our number system, probably evolved out of our 10 fingers and 10 toes. That was kind of exciting and surprising.

On the other end of the spectrum, two nontraditional participants felt mathematics to be nonstandards based. Nadine, aged 34, viewed mathematics as fixed and remarked during an interview about her frustrations with mathematics.

> I think it (mathematics) is fixed because it seems there is only one right answer and learning formulas is the way I have always learned it….With math, I like the procedural way because I don't care why….I usually get a headache when I think about why.

With the other seven participants (one nontraditional and six traditional), the researcher found the preservice teachers voiced comments that mathematics contained standards and nonstandards based ideals. Participants often discussed how mathematics could be fixed in certain situations, such as answers to mathematics questions and mathematical definitions, but that the methods to arrive at those answers could vary.

In addition to specific beliefs about mathematics, the researcher discovered an evolution of participant opinions about mathematics. She asked participants during their second set of interviews about any changes in their views about mathematics during the semester or at any other period of time in their lives. Three participants (two nontraditional and one traditional) felt all of their beliefs about mathematics remained unchanged throughout their lives, while other preservice teachers' responses often consisted of changes due to multiple reasons, such as the mathematics sequence of classes under study, family influences, experiences in the public school system, and maturity. Two nontraditional and five traditional participants attribute at least some changes in their mathematics viewpoints to the MATH 100/200/300 courses. Participants often commented on how the nonprocedural methods covered in the courses broadened their opinions about the nature of mathematics. Two nontraditional preservice teachers also discussed work with their younger family members, such as children and/or nieces and nephews, and work in the schools as opening their eyes to new opinions about mathematics. Lastly, one of the youngest nontraditional participants, Nita, remarked that age changed her negative feelings about mathematics.

Conclusions

From the findings, the researcher found that nontraditional preservice elementary teachers' views about mathematics varied more in comparison with traditional teachers. While all six traditional participants espoused to combination beliefs about mathematics, their counterparts' views varied from standards based to nonstandards based. Three nontraditional participants held standards based beliefs about mathematics; two nontraditional held nonstandards based views about mathematics; and one nontraditional preservice teacher held a combination of standards and nonstandards based beliefs about mathematics. The instructors of each MATH 100/200/300 course also held combination beliefs of mathematics, which might suggest that the younger participants may be more easily swayed in their newer belief structures than older participants who may have longer held beliefs, a finding supported by Pajares' (1992) research.

In addition to findings about belief systems, two nontraditional and five traditional preservice teachers changed their opinions about mathematics because of the MATH

100/200/300 sequence. This finding is supported by Steele's (1994) research, where she also found many preservice teachers altered their mathematics beliefs because of a mathematics methods course that focused on constructivist teaching practices.

These research findings suggest that instructors of preservice teachers may impact preservice teachers' core beliefs about mathematics, especially the younger teachers. Research needs to be conducted to see whether the belief structures are permanently influenced and transfer to the preservice teachers' actual classroom routines. Since public school experience and family connections influenced the way in which preservice teachers thought about mathematics, instructors of preservice teachers may want to provide opportunities for preservice teachers to work in the public schools and with young family members. These experiences may make the class content more meaningful to the preservice teachers and ultimately impact the preservice beliefs systems about mathematics.

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MATHEMATICS CONTENT PROFICIENCY AND BELIEFS IN THE NYC TEACHING FELLOWS PROGRAM

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The purpose of this study was to understand the mathematical content proficiency new teachers had both before and after taking a mathematics methods course in the New York City Teaching Fellows (NYCTF) program. Further, the purpose was to understand attitudes toward mathematics and teacher self-efficacy that Teaching Fellows had over the course of the semester. Findings revealed a significant increase in both mathematical content knowledge and positive attitudes toward mathematics for the Teaching Fellows. Further, Teaching Fellows were found to have generally positive attitudes and high self-efficacy at the end of the semester. Additionally, relationships were found between attitudes and self-efficacy. Finally, Teaching Fellows generally found that classroom management was the biggest issue in their teaching, and that problem solving and numeracy were the most important topics addressed in the methods course.

Introduction

Content proficiency, attitudes toward mathematics, and self-efficacy have become increasingly important issues in mathematics education (Amato, 2004; Ball, Hill, & Bass, 2005; Swars, Hart, Smith, Smith, & Tolar, 2007). These constructs were examined among two mathematics methods sections of secondary mathematics teachers in the New York City Teaching Fellows (NYCTF) program. The purpose of this study was to understand what mathematical content proficiency new teachers have both before and after a mathematics methods course, as well as what attitudes and concepts of self-efficacy these new teachers held.

Teacher content proficiency is important since it is a necessary condition for good teaching (Ball et al., 2005). Attitudes toward mathematics are important since there is a reciprocal relationship between attitudes toward mathematics and achievement in mathematics (Aiken, 1970; Evans, 2007; Ma & Kishor, 1997). Further, negative teacher attitudes toward mathematics often lead to avoidance of teaching strong mathematical content and affect students' attitudes and behaviors (Amato, 2004; Leonard & Evans, 2007). Poor attitudes toward teaching are directly related to teacher retention issues (Costigan, 2004). It has been shown that teacher self-efficacy influences student achievement (Swars et al., 2007). However, the literature has been sparse in addressing in-service secondary mathematics teachers' knowledge, attitudes toward mathematics, and self-efficacy, particularly in alternative certification and secondary education. This study expands upon the literature by examining the field experience relationship, specifically in-service teaching, and experience in a reformed-based secondary mathematics methods course with content proficiency, attitudes toward mathematics, and self-efficacy in an alternative certification program.

New York City Teaching Fellows (NYCTF) Program

The NYCTF program is an alternative certification program developed in 2000 in conjunction with The New Teacher Project and the New York City Department of Education (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007; NYCTF, 2008). The program goal was to recruit professionals from other fields to supply the large teacher shortages in New York City's public schools with quality teachers. At the outset of the program there was a 7000 teacher shortage predicted for late 2000 with a possible shortage of 25,000 teachers over the next several years (Stein, 2002). Prior to September 2003, New York State allowed teachers to obtain temporary teaching licenses to help fill the teacher shortage.

Alternative Certification Teacher Quality Literature Review

There has been a recent interest in studying the effects of alternative teacher certification in U.S. classrooms with a particular interest in teacher quality issues (Darling-Hammond, 1994, 1997; Darling-Hammond, Holtzman, Gatlin, & Heilig, 2005; Evans, 2009). Further, there has been specific interest in Teaching Fellows in New York City schools in particular (Boyd, Grossman, Lankford, Loeb, Michelli, & Wyckoff, 2006; Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007; Costigan, 2004; Kane, Rockoff, & Staiger, 2006; Stein, 2002). However, most studies investigated student achievement and teacher retention to determine teacher quality and success. Naturally these are two of the most important variables, but there is a need to investigate neglected variables in alternative certification that affect teacher quality such as teacher content proficiency, attitudes toward mathematics, and self-efficacy.

Boyd, Grossman, Lankford, Loeb, Michelli, and Wyckoff (2006) focused on student achievement and teacher retention as measures of success. Boyd et al. found that in their first year teachers prepared through alternative certification programs had students with slightly smaller achievement gains in mathematics compared with traditionally prepared teachers. For elementary teachers there were no differences found by the second year between alternatively and traditionally prepared teachers. Middle school students of Teaching Fellows performed just as well as the students of traditionally prepared teachers. By the third year of teaching, students of Teaching Fellows outperformed students of traditionally prepared teachers. However, Kane et al. (2006) found that despite strong academic credentials, a variable related to teacher effectiveness, Teaching Fellows were no more effective than their traditionally certified colleagues.

Aiken (1970) was an early pioneer in researching the relationship between mathematical achievement and attitudes toward mathematics. Like Aiken, Ma and Kishor (1997) found a small but positive significant relationship between achievement and attitudes through meta-analysis. This relationship between achievement and attitudes, along with Ball et al.'s (2005) emphasis on the importance of content knowledge for teachers, forms the theoretical framework of this study. Additionally, Bandura's (1986) construct of self-efficacy theory frames this study's focus on self-efficacy in Teacher Fellows. Bandura found that teacher self-efficacy can be subdivided into a teacher's belief in his or her ability to teach well, and his or her belief in a student's capacity to learn well from the teacher.

Research Questions

1. What differences existed between Teaching Fellows' mathematical content proficiency before and after a mathematics methods course?
2. What differences existed between Teaching Fellows' attitudes toward mathematics before and after a mathematics methods course? Further, what level of attitudes toward mathematics did Teaching Fellows possess at the end of the semester?
3. What differences existed between Teaching Fellows' concepts of self-efficacy before and after a mathematics methods course? Further, what level of self-efficacy did Teaching Fellows possess at the end of the semester?
4. Was there a relationship between Teaching Fellows' attitudes toward mathematics and concepts of self-efficacy?
5. What level of content proficiency did Teaching Fellows possess?
6. What were Teaching Fellows' attitudes toward teaching and learning mathematics?

Methodology

The methodology of this study involved both quantitative and qualitative methods. The sample in this study consisted of 42 new in-service teachers in the Teaching Fellows program enrolled in two reformed-based mathematics methods sections with approximately one third of participants male and two thirds participants female. Teaching Fellows were given a mathematics content proficiency test and two questionnaires at the beginning and the end of the semester. The mathematics content test consisted of 25 free response items ranging from algebra to calculus. The test taken at the end of the semester was similar in form and content to the one taken at the beginning. An additional mathematics content test that consisted of 15 multiple choice items, taken from the sample Content Specialty Test (CST) in mathematics from the New York State teacher certification website, was given at the beginning of the semester. Additionally, students' actual CST scores were recorded as another measure of mathematical content proficiency. Students who begin teaching in September take the CST generally in the summer before beginning the program.

The first questionnaire was from Tapia (1996) and had 40 items that measured attitudes toward mathematics including self-confidence, value, enjoyment, and motivation in mathematics. The instrument used a 5-point Likert scale ranging from strongly agree, agree, neutral, disagree, to strongly disagree. The second questionnaire was adapted from the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) developed by Enochs, Smith, and Huinker (2000), and measured teacher self-efficacy. The MTEBI is a 21-item 5-point Likert scale instrument with choices ranging from strongly agree, agree, uncertain, disagree, to strongly disagree, and was grounded in the theoretical framework of Bandura's self-efficacy theory (1986). The MTEBI contains two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE) with 13 and 8 items, respectively. Possible scores range from 13 to 65 on the PMTE, and 8 to 40 on the MTOE. The PMTE specifically measured a teacher's self-concept of his or her ability to effectively teach mathematics well, while the MTOE specifically measured a teacher's belief in his or her ability to directly affect student learning outcomes despite external factors in the students' lives.

Teaching Fellows were also required to keep reflective journals on their teaching and learning over the course of the semester, which provided qualitative data on their attitudes toward teaching and learning mathematics. The teaching journal was used as a reflection upon the Teaching Fellows' actual teaching and classroom experiences. The learning journal was used as a reflection of what was being learned in the mathematics methods course.

Results

The first research question was answered using the 25-item mathematics content test, and data were analyzed using a paired samples t-test. The results of the paired samples t-test (two-tailed) revealed a statistically significant difference between pretest scores ($M = 74.79$, $SD = 17.605$) and posttest scores ($M = 84.48$, $SD = 14.225$) for the mathematics content test with $t(41) = -6.002$, $p < 0.001$, $d = 0.86$. This means there was a statistically significant increase in content proficiency as measured by the 25-item mathematics content test over the course of the semester. Additionally, there was a large effect size.

The second research question was answered using the 40-item attitudinal test. Data were analyzed using a paired samples t-test. The results of the paired samples t-test (two-tailed) revealed a statistically significant difference between pretest scores ($M = 3.25$, $SD = 0.373$) and posttest scores ($M = 3.33$, $SD = 0.410$) for the mathematics attitudinal test with $t(41) = -2.041$, $p < 0.05$, $d = 0.20$. This means there was a statistically significant increase in positive attitudes toward mathematics as measured by the 40-item attitudinal test over the course of the semester. However, the effect size was small.

Further, the second part of the second research question was answered using an independent samples t-test. The independent samples t-test was conducted to determine if the participants had significantly better attitudes toward mathematics at the end of the semester as compared to a neutral value coded as “2” on the survey sheet. Likert scores for strongly disagree, disagree, neutral, agree, and strongly agree were coded from 0 to 4. The results of the independent samples t-test (two-tailed) revealed a statistically significant difference between attitudinal scores ($M = 3.33$, $SD = 0.410$) and neutral scores ($M = 2.00$, $SD = 0.000$) with $t(41) = 21.109$, $p < 0.001$ (equal variance not assumed), $d = 4.89$. This means that the participants had statistically significant better attitudes toward mathematics than a neutral value of “2”, and the effect size was very large. It should be noted, however, that comparing actual attitudinal scores with neutral responses should be interpreted with caution.

The third research question was answered using the 21-item MTEBI with two subscales: PMTE and MTOE. Data were analyzed using paired samples t-tests. The results of the paired samples t-test (two-tailed) revealed no statistically significant difference between pretest scores ($M = 2.90$, $SD = 0.435$) and posttest scores ($M = 2.94$, $SD = 0.486$) for the PMTE with $t(41) = -0.551$, $p = 0.584$. This means there was no increase in belief in self-efficacy toward teaching as measured by the PMTE over the course of the semester. Further, the results of a second paired samples t-test (two-tailed) revealed no statistically significant difference between pretest scores ($M = 2.73$, $SD = 0.481$) and posttest scores ($M = 2.74$, $SD = 0.505$) for the MTEO with $t(41) = -0.170$, $p = 0.866$. This means there was no increase in belief in affecting student outcomes as measured by the MTOE over the course of the semester.

Further, the second part of the third research question was answered using independent samples t-tests. Independent samples t-tests were conducted to determine if the participants had significantly better concepts of self-efficacy at the end of the semester as compared to a neutral value coded as “2” on the survey sheet. For the PMTE the results of an independent samples t-test (two-tailed) revealed a statistically significant difference between PMTE scores ($M = 2.94$, $SD = 0.486$) and neutral scores ($M = 2.00$, $SD = 0.000$) with $t(41) = 12.565$, $p < 0.001$ (equal variance not assumed), $d = 2.73$. This means that the participants had statistically significant better attitudes toward mathematics than a neutral value of “2”, and the effect size was very large. For the MTOE the results of an independent samples t-test (two-tailed) revealed a statistically significant difference between MTOE scores ($M = 2.74$, $SD = 0.505$) and neutral scores ($M = 2.00$, $SD = 0.000$) with $t(41) = 9.513$, $p < 0.001$ (equal variance not assumed), $d = 2.07$. This means that the participants had statistically significant better beliefs to affect student learning outcomes than a neutral value of “2”, and the effect size was very large.

Pearson correlations were used to answer research question four. It was found that there was a statistically significant correlation between pretest mathematics attitude scores ($M = 3.25$, $SD = 0.373$) and pretest PMTE scores ($M = 2.90$, $SD = 0.435$) with $r = 0.690$, $n = 42$, and $p < 0.001$. Additionally, it was found that there was a statistically significant correlation between posttest mathematics attitude scores ($M = 3.33$, $SD = 0.410$) and posttest PMTE scores ($M = 2.94$, $SD = 0.486$) with $r = 0.491$, $n = 42$, and $p < 0.01$. No correlation was found between mathematics attitude scores and MTOE scores.

The fifth research question was answered using the 15-item sample Content Specialty Test (CST) and student scores on the actual CST that students take for New York State certification. On the 15-item sample CST the mean score was 10.38 and standard deviation was

3.012. The potential range of scores is 0 to 15. The mean score for actual CST scores was 260.62 and standard deviation was 20.184. The passing score required in New York State for the CST is 220 and the highest possible score is 300. A Pearson correlation was used to determine the relationship between these scores. A statistically significant correlation was found between the two examinations with $r = 0.529$, $n = 42$, $p = 0.000$. This means that the sample CST test given at the beginning of the semester was directly related to the CST taken previous to beginning the course. Interestingly, several students stated that the sample CST questions taken from the New York State website are more difficult than the actual CST. This is reflected in the fact that on the 15-item sample CST the average score was only 69.2 out of 100. However, on the actual CST the average score of 260 is well above the passing score of 220.

The sixth research question was answered using Teaching Fellows' teaching and learning journals. Analysis of the teaching journals revealed that the most commonly addressed topic was classroom management. Several Teaching Fellows mentioned that classroom management was not as much an issue as they thought it would be. However, most of the Teaching Fellows believed that classroom management issues were of their highest concern. Two Teaching Fellows were physically threatened by students, which was of great concern for both of them. One student felt that she was unprepared to deal with the classroom management issues that she encountered. After classroom management, student motivation and attendance issues were also addressed by the Teaching Fellows. Further, students' lack of basic skills, collaborative learning in the classroom, time management issues, and student lack of conceptual understanding were issues addressed in the teaching journals. Overall, it appeared that Teaching Fellows generally believed their classroom experiences were good and that they were able to positively affect student learning. The results of research question three reported that PMTE and MTOE scores did not increase over the course of the semester. However, it was found that at the end of the semester Teaching Fellows had high self-efficacy. This finding was further triangulated with the teaching journals.

Analysis of the learning journals revealed that the most commonly addressed topics were problem solving and numeracy. Since the reformed-based mathematics methods course was taught from a problem solving perspective Teaching Fellows were given a "problem of the day" that they solved collaboratively at the beginning of each class. Problem solving as a way of teaching was thoroughly addressed in the course with considerable time devoted to problem solving in teachers' classrooms. It should be noted that one student stated that even though he enjoyed the problem solving aspect of the course, he felt that too much time may have been spent on it. Further, in addition to the mathematics methods textbook (Posamentier, Smith, & Stepelman, 2006), Teaching Fellows read Paulos' *Innumeracy: Mathematical Illiteracy and its Consequences* (1990). In this book Paulos addressed what it means to be numerate (i.e. mathematically literate) in a democratic society. Furthermore, Teaching Fellows thought microteaching and motivation techniques enhanced their learning in the course. Every Teaching Fellow was required to present a brief microlesson that contained a motivator intended to gain student interest. Since teachers felt they had trouble motivating their students, many found microteaching and general motivational techniques covered in this course to be helpful. One student mentioned that at times the course was more theoretical and less practical than she would have preferred. This is consistent with findings from Costigan (2004).

Discussion

It was found that Teaching Fellows increased their mathematical content proficiency over the course of a one semester reformed-based mathematics methods course while teaching in their own classrooms. Additionally, it was found that Teaching Fellows had an increase in positive attitudes toward mathematics over the course of the semester, and at end of the semester they generally held positive attitudes toward mathematics. It is speculated that a reformed-based mathematics methods course with an emphasis on mathematics for understanding and problem solving, combined with in-service teaching experiences, contributed to the growth of teacher content proficiency and better attitudes toward mathematics (Palmer, 2006). Content proficiency in secondary level teaching is important since the mathematics required is more sophisticated than at the elementary level. Prior research primarily focused on the elementary level. Since Teaching Fellows come to the profession without mathematics majors in many cases, content proficiency is of particular concern. The results of this study should help alleviate some of the concern that alternatively certified teachers are unprepared to teach since it has been shown that a reformed-based methods course and in-service teaching can lead to immediate growth in content proficiency and positive attitudes toward mathematics as measured in this study.

No increase was found in teacher self-efficacy on both the Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcomes Expectancy (MTOE) subscales. However, it was found that at the end of the semester Teaching Fellows generally had high concepts of self-efficacy both in terms of their ability to teach well (as measured by the PMTE), as well as their ability to positively affect student outcomes (as measured by the MTOE). Additionally, evidence from Teaching Fellows' journals further indicated high concepts of self-efficacy. Since teachers already had high self-efficacy it is not very surprising that there were not significant increases in self-efficacy. However, these results are inconsistent with the literature and thus inconclusive (Palmer, 2006; Swars et al., 2007). Findings in the literature suggested that over the course of a methods course teacher efficacy increased. Further, Hoy and Woolfolk (1990) reported that teacher outcome expectancy declined when pre-service teachers began teaching. However, teacher outcome expectancy in the literature had been examined for teachers transitioning from pre-service to in-service. The participants in this study began as in-service teachers, which means they encountered the realities of the classroom immediately. The self-efficacy aspect for in-service teaching should be further investigated, particularly at the secondary level.

A positive correlation was found between Teaching Fellows' attitudes toward mathematics and PMTE scores for both pre and posttests. However, no relationship was found for attitudes toward mathematics and MTOE. This is consistent with the literature (Swars, Daane, & Giesen, 2006) when examining the relationship between mathematics anxiety and self-efficacy using the PMTE and MTOE subscales. Mathematics anxiety has been shown to be related to attitudes toward mathematics (Ma, 1999). This study indirectly and partially confirms the findings of Swars et al. (2006) at the secondary level for in-service teachers.

It was found that Teaching Fellows generally stated that classroom management was the biggest issue in their teaching, and that problem solving and numeracy were the most important issues addressed in the methods course. It was not surprising that teachers found classroom management to be the biggest issue since this is consistent with the literature for new teachers (Costigan, 2004; Cruickshank, Jenkins, & Metcalf, 2006; Veenman, 1984). However, it was surprising that several teachers stated that classroom management was not an issue for them. This is in contrast to contrary findings with Teach for America alternative certification (Evans, 2009), in which classroom management was exclusively problematic. Finally, it was expected that teachers would find problem solving and numeracy to be two of the most valuable topics addressed in the course since there was a strong emphasis placed on both in the course.

The results of this study contribute to the literature since teachers demonstrated high self-efficacy at the end of the mathematics course. Considering the participants were in-service teachers, this has interesting implications about the teacher preparatory program since the literature shows teachers tend to have higher levels of student outcome expectancy while they were pre-service teachers. However, outcome expectancy generally declines when the teachers become in-service and the realities of the classroom are encountered (Swars et al., 2007). Secondary alternative certification teachers' student outcome expectancy should be further investigated in future research. Also, future studies should determine if attitudes toward mathematics and concepts of self-efficacy are durable and sustained over time.

It is hoped that there will continue to be more studies at the secondary level on alternative certification, specifically in the NYCTF mathematics program. Understanding teachers' mathematics content proficiency, attitudes toward the subject, and self-efficacy is important for professors of education to guide teacher education instruction as well as provide much needed support for new teachers. This is more critical now than ever considering the ever increasing pool of New York mathematics teachers who enter the profession through the Teaching Fellows program and other alternative certification programs elsewhere throughout the United States. Teacher quality in alternative certification has a direct impact on the many urban students who receive these new teachers in their classrooms. Given the rhetoric in education regarding equity and social justice, more studies are necessary on this unique group of teachers who teach urban students.

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THE EFFECTIVE DIFFERENCE OF RESEARCH PROJECTS ON SECONDARY MATHEMATICS PRESERVICE TEACHERS' SENSE OF EFFICACY

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The purpose of this quantitative study was to investigate the difference in teacher efficacy measures of two groups of preservice teachers who were given modified research projects and were enrolled in a secondary mathematics methods course. The participants were divided into two groups doing modified research projects related to the field of mathematics education. The modification of research projects were grounded in one of Bandura's (1997) sources of self-efficacy: vicarious experience. Two possible vicarious experiences that inform preservice teachers' sense of teacher efficacy are reading professional literature and watching others teach followed by discussing the results. These two contexts are the basis of the research projects modifications. Data revealed that there were statistically significant differences between the two groups' teacher efficacy measures. Those who did the research project involving observations and discussion of mathematics teaching had significantly higher measures of teacher efficacy over those who did their research purely through professional literature.

Introduction

The engagement and preparation of preservice teachers in the profession is of vital importance since preservice experiences can have significant consequences as graduates face their first five years of teaching. Educators need to examine what experiences are needed, how these experiences are offered, and why they are valuable. The mathematics education community wrestles with higher teacher attrition rates more than other fields (Guarino, Santibanez, & Daley, 2006), with reports of high stress levels among teachers, and with the surge to produce teachers that are highly qualified. One part of solving these widespread difficulties could be determining and implementing ways of engaging preservice teachers that act to increase their teacher efficacy.

Tschannen-Moran, Hoy, and Hoy (1998) report that the research has established a relationship between teacher efficacy and "teachers' classroom behaviors, their openness to new ideas, and their attitudes on teaching. In addition, teacher efficacy appears to influence student achievement, attitude, and affective growth (p. 10)." Furthermore, it has been found that higher levels of teacher efficacy connect to lower levels of teacher stress. Because of this relationship between teacher efficacy and stress, it has been suggested that the education community work to increase teacher efficacy as a solution to stress and teacher burnout (Parkay, Greenwood, Olejnik, & Proller, 1988).

While many relationships have been discovered between teacher efficacy and elements of education, there are very few studies aimed at determining what types of experiences and academic engagements influence the growth in teacher efficacy of preservice teachers. Although studies have been done examining teacher efficacy growth during mathematics methods courses

(Utley, Moseley, & Bryant, 2005), after methods courses (Huinker & Madison, 1997), and before and after clinical and student teaching experiences (Utley et al., 2005; Vinson 1995) there is little research that examines with great specificity what is done during these times in a teacher's development that would promote higher levels of teacher efficacy. While it is known that mathematics methods courses contribute to a preservice teacher's growing sense of teaching efficacy, what elements of those courses contribute most to teacher efficacy? It is important to consider particular preservice teacher experiences and examine the effect on teaching efficacy. This study is a focused examination of the relationship between preservice teachers' efficacy and two types of research assignments given in methods courses.

Literature Review and Theoretical Framework

Teacher efficacy has been defined as the extent to which teachers believe they can strongly influence student achievement and motivation in learning (Ashton, 1985; Tschannen-Moran, Hoy, & Hoy, 1998). For a little more than three decades educational researchers have been working to define the construct of teacher efficacy, clarify its conceptual underpinnings, and measure its relationships. An historical accounting of teacher efficacy understanding follows.

The construct of teacher efficacy has its theoretical beginnings in Rotter's (1966) social learning theory. Rotter's work was the inspiration for a small part of a study done by the Rand Corporation (Armor, Conroy-Oseguera, Cox, King, McDonnell, Pascal, Pauly, & Zellman, 1976) in which they measured teacher efficacy by summing scores of two items on a survey. The first item asked teachers whether environmental and motivational factors of students could be overcome by teachers, as a general group, measuring what is now referred to as general teaching efficacy (GTE). The second item asked, from the first person perspective, about the degree to which the teacher was confident in getting through to the most difficult students, measuring what is now referred to as personal teaching efficacy (PTE). Throughout the 1980's and 1990's teacher efficacy was further influenced by Bandura's social cognitive theory (Bandura 1977, 1986, 1993, 1997).

In 1984, Gibson and Dembo applied Bandura's psychological construct of self-efficacy to the teaching field and foresaw that teachers' sense of efficacy could account for variations in teaching ability. Bandura defined self-efficacy as a person's judgment of how well he or she could perform an action to deal with a situation. He claimed that when one has low self-efficacy, less effort might be given and one will encounter more stress from the demands of having to perform the action. When applied to the act of teaching, efficacy is more specifically thought of as the teacher's beliefs about his or her ability to influence student learning. These beliefs can affect the amount of effort a teacher gives and the amount of stress a teacher encounters.

From these theoretical bases, research on teacher efficacy has been found to have significant influence on teacher practice and student learning (Smith, 1996). Early research found a positive correlation between a teacher's sense of efficacy and whether or not the teacher stayed in the field (Glickman & Tamashiro, 1982), the amount of teacher change and project methods integrated into the classroom from grant workshops teachers attended (Berman, McLaughlin, Bass, Pauly, Zellman, 1977), a teacher's production of higher measures of student achievement (Allinder, 1995; Ashton & Web, 1986), a teacher's persistence in working with struggling students (Ashton & Webb, 1986; Gibson & Dembo, 1984), and willingness to try innovative curriculum (Guskey, 1988).

As efficacy research grew, evidence and refinements to the construct indicated a necessity to look more closely at the role played by the context and subject matter as well as the appropriate level of specificity for measuring teacher efficacy (Tschannen-Moran et al., 1998). Furthermore, it is important to understand the effects of preservice teacher training on teacher efficacy and what aspects appear to be rigid or malleable in a particular subject domain. Reliable and valid instruments were made in mathematics and science to better investigate subject matter specific teacher efficacy (Enochs & Riggs, 1990; Enochs, Smith, P., & Huinker 2000). Using these content specific instruments researchers have found that preservice teachers' sense of personal efficacy and outcome expectancy increased significantly in science when taking an integrated mathematics/science course while those students in a non-integrated course had no change (Moseley & Utley, 2006). Another study by Utley, Moseley, and Bryant (2005) showed an increase in teaching efficacy as preservice teachers participated in mathematics methods coursework but a slight decline in teaching efficacy by the end of student teaching.

More often than not, research has supported Gibson and Dembo's (1984) prediction that teachers who continue to wrestle with the difficulties of the teaching profession have high measures of general and personal teaching efficacy, while those with low measures do not persist and often leave the profession. Teaching efficacy has been connected with what mathematics the teachers teach and what their students end up learning (Peterson et al., 1989). Furthermore, low teaching efficacy acts as a factor in preservice teachers' reluctance to teach mathematics (Wenner, 2001). Determining what kinds of professionally engaging tasks to give to preservice teachers to allow for growth in their teaching efficacy is important, yet remains under researched. Knowledge about such tasks can inform education programs about better equipping preservice teachers for a longer and more fruitful duration in the profession.

Purpose

Bandura (1986, 1997) suggested four broad categories by which knowledge of the act of teaching and self-perceptions of teaching are constructed. Preservice and in-service teachers use mastery experiences, physiological and emotional states, vicarious experiences, and verbal persuasion to inform self-efficacy beliefs. While all four contribute to preservice teachers developing beliefs of competence for the task of teaching mathematics, *vicarious experiences* were used as the source of comparison for the research tasks in this study. Vicarious experiences are those in which a teacher or preservice teacher is informed about the teaching task by observing others teach, reading professional literature, or engaging in tasks given in teacher education courses. From these experiences preservice teachers develop ideas about what the results of successful teaching look like, what actions lead to successful outcomes, and whether or not they are capable of such actions. These ideas led to the research question: In what kinds of tasks and vicarious experiences can preservice teachers be engaged that will encourage them to develop a strong sense of personal and general teaching efficacy?

A common vicarious experience given in preservice course work is the assignment of research papers meant to allow preservice teachers to use professional literature to inform themselves about the act of teaching, its measures, and attributes. The purpose of this study is to investigate whether or not there is a difference in the teaching efficacy of two groups of mathematics preservice teachers which were given different research tasks both oriented toward the overcoming of difficult teaching situations.

Method

Participants and Study Sampling

The population of the study was undergraduate students majoring in middle level or secondary mathematics education and attending a university in the mid-south United States. All sixteen (n = 16) participants were enrolled in the same mathematics methods course. The primary investigator served as the instructor in the mathematics course.

Throughout the mathematics methods course the participants were all given the exact same assignments, except one. The assignment of exception was a research project from which participants had two options:

a) *Text-Based Research* - Starting from a literature review on the teaching of mathematics in schools located in urban areas of poverty, each student chose a topic and prepared a paper on how mathematics teachers are best adapting to overcome difficulties encountered in this school context. From this research they discussed actions they would take (and why) to ensure significant mathematics learning in their classroom.

b) *In-Field Research* - Starting from a literature review on the teaching of mathematics in schools located in urban areas of poverty, students designed a set of interview questions for mathematics teachers from which to analyze and produce a paper on how mathematics teachers are best adapting to overcome difficulties encountered in this school context. From this research they discussed actions they would take (and why) to ensure significant mathematics learning in their classroom. For this research the participants missed three days of class to travel out of state to visit, observe, and interact with teachers and students at an urban middle and high school located in an area of poverty. The interview questions were asked by the participants to teachers at the urban school after observations of teaching and interactions with students.

At the beginning of the research task all sixteen participants were asked to brainstorm issues they believed teachers of mathematics in urban areas of poverty might face. From this exercise the participants came up with 27 issues which they put into 5 categories: working with diversity, discouraging truancy and dropping out, motivating teaching strategies, accounting for mathematical deficiencies, and overcoming apathy in the mathematics classroom. Each participant then chose a different topic located within a category to further research, chose two articles in the literature over their topic, and shared the findings with their peers.

All participants were given the opportunity to complete the project as either an in-depth text-based research project or attend the in-field research project. Participants self-selected the way they would complete the project based on their own school schedules and life obligations.

This resulted in eight participants choosing the text-based research paper and the other eight choosing the in-field research. Those doing the in-depth text based research completed an exhaustive study on their topic about what is known about effective teaching in this context and what they would apply to their own teaching and why. Those doing the in-field research worked to create interview questions for mathematics teachers, conduct the interviews, and then analyze the findings for the paper. While both groups of students started from the same set of categories and base literature review, those doing the field research spent additional time learning about interviewing as a research tool.

Urban School Context for In-Field Research

Due to the requirements of the research project a public school in an urban area of poverty was chosen for the research trip. This school was chosen not only for the challenges presented by serving students in urban poverty but also for the history of positive results of its mathematics faculty in dealing with these issues and their high sense of collective efficacy. Participants could use this experience to increase understanding of systemic approaches to overcoming a challenging teaching environment.

The school is an inner-city school located in a metropolitan area of more than 1.2 million residents. The school is located in a section of the city with a historically low economic level as indicated by an 86% free and reduced lunch rate. The ethnic diversity at the school from largest population to lowest is 73% Hispanic, 15% Caucasian, 7% Native American, and 5% African American. The school is noted for its creative scheduling to discourage truancy issues and allow for flexibility in teaching. The school has a creative program to assist highly at-risk students and dropouts in finishing their education and receiving a diploma. The school out-performs its neighboring schools academically by significant margins while at the same time has cut the percentage of dropouts to nearly half the number of students that drop out of neighboring schools.

The faculty at the school has a strong collective teaching efficacy. Educators believe that this is partly the result of the teachers being allowed to participate in decision-making at the school, often receiving positive feedback from peers, and the principals' strong leadership which constantly encourages innovation to push the students to better learn mathematics. The effect of these characteristics is supported by several studies done on teacher efficacy. According to Bandura (1993), how a school performs academically is related to the teacher's collective beliefs in their instructional efficacy in that the stronger the collective belief the greater the results academically. In turn, a school's collective sense of efficacy was shown to be higher when principals were perceived as caring and encouraged innovation (Newmann, Rutter & Smith, 1989). Higher general teaching efficacy occurs when principals inspire a common sense of purpose (Hipp & Bredeson, 1995). Higher personal teaching efficacy has been found among teachers who felt they have influence in school-based decisions (Moore & Esselman, 1992). The school in the context of this study has principals who exhibit these characteristics and a school ecology which nurtures high teacher efficacy.

Teaching Efficacy Instrument

The instrument used in this investigation was the Teacher Efficacy Scale Short Form (Hoy & Woolfolk, 1990). The instrument consists of ten items which can be found in Appendix A.

Respondents rated each item on a six point Likert-scale. On questions 1, 2, 4, 5, and 10 responses were assigned a number from 1 (Strongly Agree) to 7 (Strongly Disagree), whereas the scale for the remaining questions was reversed so that a higher score consistently corresponds to a higher sense of efficacy. Questions 3, 6, 7, 8, and 9 are pooled together to measure personal teaching efficacy (PTE) and the remaining five questions are pooled together to measure general teaching efficacy (GTE). This short form resulted from modifications of Gibson and Dembo's (1984), 30-item measure of teacher efficacy where only 16 of the items produced adequate reliability, further reduced to ten items to eliminate cross loading of PTE and GTE items while still retaining the appropriate measure. Hoy and Woolfolk (1993) used this shortened form and found reliabilities of alpha 0.77 for PTE and 0.72 for GTE.

Results

Minitab was used to generate stacked dot plots; see Figures 1 and 2. The individual responses are separated by personal teaching efficacy and general teaching efficacy and grouped by the two groups of students: those performing the field experience research and those who perform the text based research.

A visual inspection of the plots indicates that three of the graphs have data that is fairly symmetric, but the dot plot of GTE by those with the field experience was somewhat skewed to the left. More importantly, there appears to be a significant difference in the center of the two groups in both GTE and PTE.

Figure 1. GTE Stacked Dot Plots of Individual Responses

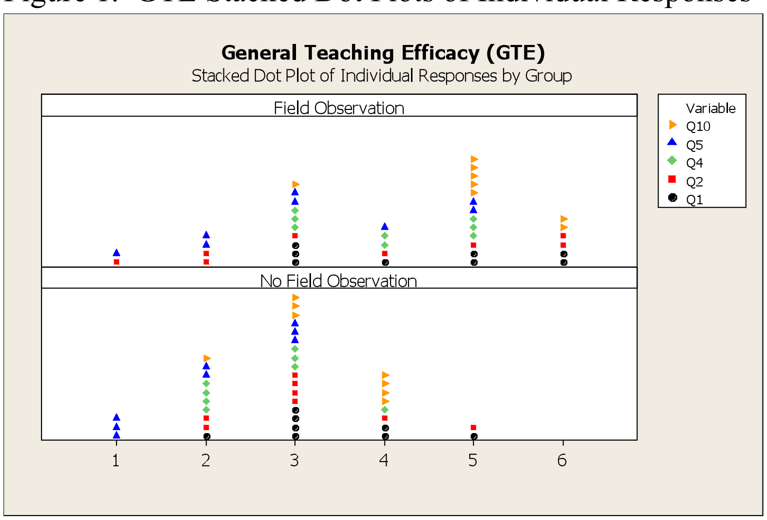
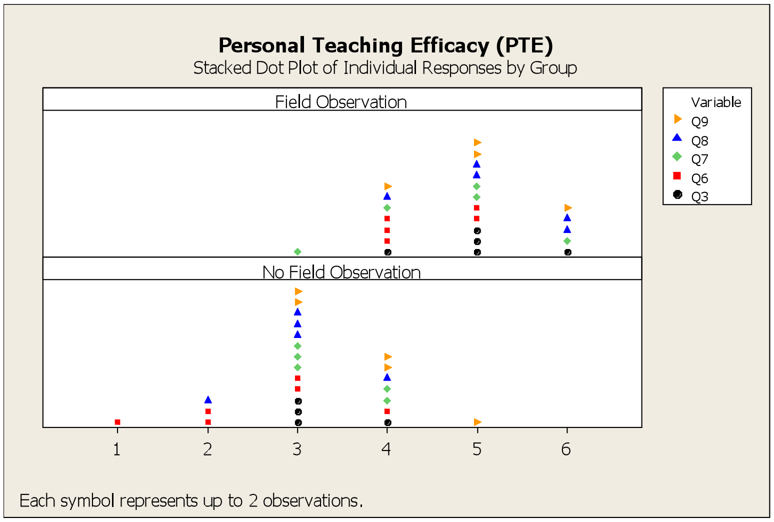


Figure 2. PTE Stacked Dot Plots of Individual Responses



The data was examined further by determining the mean and median for each of the two groups on each individual question and on the GTE and PTE groups of questions. A series of one-tailed *t*-tests were used to compare means. Since the normality requirement of these tests was suspect a series of one-tailed Mann-Whitley tests with correction was performed for ties. This test is used to compare medians and is independent of the underlying distribution. The measurements and *p*-values from these analyses are included in Table 1.

Table 1.

Comparison of Means and Medians of the Groups by Question

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**GTE**	**PTE**
Mean No Field Experience	3.375	3.125	3.250	2.625	2.000	2.500	3.375	3.125	3.750	3.375	2.900	3.200
Mean With Field Experience	4.375	3.625	5.125	4.000	3.125	4.375	4.750	5.125	5.000	5.000	4.025	4.875
Difference in Means	**1.000**	**0.500**	**1.875**	**1.375**	**1.125**	**1.875**	**1.375**	**2.000**	**1.250**	**1.625**	**1.125**	**1.675**
***t*-test *p* value**	**0.050**	**0.264**	**0.000**	**0.003**	**0.045**	**0.000**	**0.003**	**0.000**	**0.002**	**0.001**	**0.000**	**0.000**
t											**-4.08**	**-9.67**
Median No Field Experience	3.000	3.000	3.000	2.500	2.000	2.500	3.000	3.000	4.000	4.000	3.000	3.000
Median With Field Experience	4.500	3.500	5.000	4.000	3.000	4.000	5.000	5.000	5.750	5.750	4.000	5.000
Difference in Medians	**1.500**	**0.500**	**2.000**	**1.500**	**1.000**	**1.500**	**2.000**	**2.000**	**1.750**	**1.750**	**1.000**	**2.000**
Mann-Whitney Test *p* value	**0.078**	**0.357**	**0.001**	**0.008**	**0.071**	**0.001**	**0.008**	**0.001**	**0.007**	**0.003**	**0.000**	**0.000**

Regardless of the test used the same conclusion was reached. In every case the measured means and medians are higher for the group of preservice teachers who completed the field experience research project. At the a = 0.10 level these differences are statistically significant for the GTE and PTE groups of questions and for each of the individual questions except for question number 2. This data supports the conclusion that the field experience research project had a significant positive effect on both general and personal measures of teacher self-efficacy.

Implications and Discussion

These research outcomes point to possible benefits that might result from intentionally designing programs that engage preservice teacher candidates in ways that increase their personal teaching efficacy and general teaching efficacy. The field of mathematics education has need of training and retaining effective mathematics teachers. The data suggests that, when possible, preparers of preservice teacher programs should incorporate research projects in which preservice teachers engage in and with teachers in highly effective schools that are overcoming considerable challenges. In addition to its potential to raise the preservice teachers' efficacy, it allows for numerous other learning opportunities and attitudinal inspiration.

While there was a significant difference between the efficacies of the preservice candidates who did the in-field research over those who did not do the in-field research, this experience is only one of many vicarious experiences preservice teachers have throughout their course work. Due to the relatively small sample size the mathematics education community could benefit from similar research done by others in various locals.

The preservice teachers who did their research "in the field" tended to have more positive comments about their learning from the assignment. This group of preservice teachers also gave vastly more descriptive reflections about their learning and spoke with more confidence during university research presentations. One of the differences between the two types of research projects involves the idea of "collective efficacy." The group of students who went together to investigate how the teachers at a public school overcame difficulties to teach mathematics to their students bonded in ways that those who worked alone on the text-based project did not. Throughout the field experience trip the preservice teachers had multiple opportunities to discuss findings and share exciting observations.

Another difference in the research projects that might account for the variability of teacher efficacy between the two groups is their physiological and emotional states (Bandura 1997). During the in-field research trip preservice teachers were constantly thrust into various states of emotion as they helped teachers and students, were told success stories by principals, teachers, and students, and became more knowledgeable about societal inequities. At times some of the preservice teachers were exhilarated about something they saw and at other times crying about a story involving a student who overcame great odds. Further research is needed on the role these experiences play, if any, on a preservice teacher's efficacy.

Working to find educational contexts that work to nurture preservice mathematics teachers' sense of efficacy can help teacher education programs form and assess various experiences that result in better prepared and more confident teachers who are more willing to stay in the field when they encounter difficult situations.

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Appendix A
Teacher Efficacy Scale Short Form (Hoy & Woolfolk, 1990)

1. *The amount a student can learn is primarily related to family background.*
2. *If students aren't disciplined at home, they aren't likely to accept any discipline.*
3. *When I really try, I can get through to most difficult students.*
4. *A teacher is very limited in what he/she can achieve because a student's home environment is a large influence on his/her achievement.*
5. *If parents would do more for their children, I could do more.*
6. *If a student did not remember information I gave in a previous lesson, I would know how to increase his/her retention in the next lesson.*
7. *If a student in my class becomes disruptive and noisy, I feel assured that I know some techniques to redirect him/her quickly.*
8. *If one of my students couldn't do a class assignment, I would be able to accurately assess whether the assignment was at the correct level of difficulty.*
9. *If I really try hard, I can get through to even the most difficult or unmotivated students.*
10. *When it comes right down to it, a teacher really can't do much because most of a student's motivation and performance depends on his or her home environment.*

THE WORKING MEMORY DEMANDS OF SIMPLE FRACTION STRATEGIES

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The present study examined the roles of phonological and visuo-spatial working memory resources in adults' strategies for comparing the sizes of simple fractions. A dual-task experiment with the choice/no-choice method was used to independently analyze the effects of working memory load (phonological or visuo-spatial) on strategy selection and strategy execution in a fraction comparison task. Load effects for both phonological and visuo-spatial working memory were found, although a concurrent visual working memory load impaired the execution of the fraction comparison task more than did a concurrent phonological load. In addition, selective involvement of working memory as a function of strategy type was found. Conceptual strategies were less affected by concurrent working memory load than were procedural strategies.

Working memory, the ability to store and manipulate information in the short term, is one of the basic functions of human cognition. Perhaps not surprisingly, working memory is vital when people are engaged in a wide variety of complex mathematical tasks (DeStefano & LeFevre, 2004). However, the extent to which the storage and rehearsal functions of working memory are employed depends on the nature of the mathematical task and the specific solution strategy that is used (Hecht, 2002; Imbo & Vandierendonck, 2007a, 2007b). In the present research, I examined the role of working memory in both conceptual and procedural fraction comparison strategies.

According to the working memory model of Baddeley and Hitch (1974) (see also Baddeley, 2007), working memory consists of four interdependent subsystems: the central executive, phonological loop, visuo-spatial sketchpad, and episodic buffer. The central executive is a limited capacity system that is responsible for control, monitoring, response selection, updating, sequencing, and planning. The phonological loop and visuo-spatial sketchpad are secondary systems that allow for the storage and rehearsal of phonological and visuo-spatial information, respectively. The episodic buffer is a system that combines the short-term function of working memory with information from long-term memory.

Previous research has indicated that the phonological loop may be used in complex mental arithmetic to store intermediate results, such as partial sums or products (Ashcraft, 1995). Indeed, recent studies investigating specific mental computational strategies have indicated that people exhibit performance decrements (such as slower reaction times or increased error rates) when doing arithmetic with a nonretrieval strategy while simultaneously holding phonological information in working memory (Imbo & Vandierendonck, 2007a, 2007b). Similar effects of concurrent phonological load have been found for complex mental multiplication (Trbovich & LeFevre, 2003). This lends support to the prediction that people may use phonological working memory resources in a fraction comparison task, particularly when engaged in procedural strategies such as cross-multiplication. It is not yet clear whether phonological resources are necessary for fraction comparison with a more holistic, conceptual strategy, such as benchmarking to common fractions.

The role of the visuo-spatial sketchpad in complex mental arithmetic is less clear. To date, significant visuo-spatial load effects have only been found for vertically-presented 2-digit by 1-digit multiplication problems (Trbovich & LeFevre, 2003) and horizontally- and vertically-presented 2-digit subtraction problems (Imbo & LeFevre, in press). In recent neuroimaging work, Ischebeck, Schocke, and Delazer (2009) found increased activity in the intraparietal sulcus (IPS) when adults were engaged in a fraction comparison task. Along with evidence for the role of the IPS in visuo-spatial working memory (Todd & Marois, 2004), it is possible that people may use visuo-spatial working memory resources in fraction comparison, both in procedural strategies (visually keeping track of intermediate computational results on a mental blackboard) and conceptual strategies (relying on visuo-spatial representations of the two fractions).

The current experiment uses a dual-task method combined with the choice/no-choice method (Siegler & Lemaire, 1997) to investigate the roles of phonological and visuo-spatial working memory in procedural and conceptual strategies for fraction comparison. The choice/no-choice method allows independent analysis of strategy selection and strategy efficiency.

Method

Participants

Fifty-nine undergraduate students at Texas A&M University-Commerce participated in the present experiment (42 women and 17 men). The mean age was 24.6 years (age range 18-56). Participants were selected from the subject pool maintained by the Department of Psychology and Special Education. Participants volunteered for the experiment with no prior knowledge of the tasks or goals of the experiment, lowering the possibility of selection bias based on mathematical ability.

Materials

Fraction stimuli. The stimuli consisted of 48 pairs of proper fractions, divided into 4 disjoint sets of 12. Each set of 12 was constructed by crossing the factors of (a) critical fraction (1/2, 1/3, 2/3), (b) position of the critical fraction (left/right), and (c) relative size of the critical fraction (greater/less). Care was taken to make each of the fraction sets as similar as possible with respect to various structural variables of the fraction pairs, such as the numerical distance between the two fractions and the average cross product, as these variables have been found to significantly predict reaction time (Faulkenberry, 2010; Ischebeck, Shocke, & Delazer, 2009).

Phonological load task. Phonological memory load items were constructed as a list of 48 consonant-vowel-consonant (CVC) nonwords. Across participants, the list was accessed so that each participant received a different problem/load item combination. During each trial in the load condition, participants were asked to subvocally rehearse the CVC nonword while completing a fraction comparison trial. Also, a list of 48 probe CVCs was constructed with half being the same as in the original CVC list and the other half differing from those in the original CVC list by exactly one letter. For example, if the CVC presented before the comparison task was NUQ, the probe item would have been either NUQ or NUW. Participants were asked at the end of each trial whether the probe CVC matched exactly the CVC presented at the beginning of the trial.

Visuo-spatial load task. Visual memory load items were constructed as patterns of 4 asterisks arranged in a 5 x 5 square array. Specifically, a list of 48 different 4-asterisk patterns was constructed. Care was taken to make sure that the patterns of asterisks did not resemble anything recognizable, such as a number or a letter that could be remembered by recalling verbal information. Probe items were constructed in a similar manner to the phonological load task, where non-identical probes were constructed by moving exactly one asterisk by one unit, either up, down, left, or right.

Procedure

Each participant was tested individually at a computer equipped with a button box for input. The experiment took approximately 1 hour to complete. Participants were randomly assigned to either the phonological load condition or the visuo-spatial load condition and solved 12 fraction comparison problems in each of the 6 conditions defined by the 2 (Working memory load: no load, load) × 3 (Strategy: Choice, Conceptual only, Procedural only) design. In addition, each participant completed 12 trials of the working memory load task alone. The order of the conditions was counterbalanced across participants with the exception that the choice condition always preceded the conceptual/procedural-only conditions.

Each trial began with the word READY shown in the center of the screen and displayed for 1 second. The word READY then flashed on and off twice at 500-msec intervals. At the end of the last 500-msec interval, the trial stimulus appeared and remained active until the participant responded. In the no-load condition, only a fraction pair appeared, after which the participant was asked either (a) Which strategy did you use? (choice condition) or (b) Were you able to successfully use the required strategy? (conceptual/procedural only conditions). The load task trials were identical, except that the fraction pair was preceded by a memory load item (either a CVC or a visual grid) and followed by the corresponding memory probe item.

Results

Four of the fifty-nine participants were removed from further analysis due to having error rates of 50% or above in the no-load/choice condition. Of the remaining 55 participants, 29 were in the visuo-spatial load condition, and 26 were in the phonological load condition. This resulted in a total of 4,620 trials completed. Of these trials, 264 (5.7%) included an error on the fraction comparison task, and 143 (3.1%) included a failure to use the required strategy in one of the no-choice conditions. All data were analyzed using the multivariate general linear model, and unless otherwise noted, all results were considered to be significant at the alpha = 0.05 level.

Strategy Efficiency

To analyze strategy efficiency, only response times and error scores from the conceptual/procedural-only conditions were included. For each participant, median response times were computed from the trials that included both a correct answer on the fraction comparison and a successful execution of the required strategy. In addition, combined error scores were computed for each participant. The combined error score was computed by recording a trial as an error trial if either (a) an arithmetic error was committed on the fraction comparison task or (b) an error was committed on the load task. A 2 x 2 x 2 multivariate analysis of variance was conducted on correct median RT scores and the $\sin^{-1}\left(\sqrt{p}\right)$-transformed combined error scores with working memory load type (visuo-spatial, phonological) as a between-subjects factor, and working memory load (load, no-load) and strategy (conceptual, procedural) as within-subjects factors (see Table 1).

Univariate analyses revealed no significant differences among the reaction time data. Rather, strategy efficiency effects were found in the combined error scores. The scores were higher for participants in the visuo-spatial load condition (11.3%) than in the phonological load condition (6.46%), $F(1, 53) = 5.35$, partial $\eta^2 = 0.092$. Combined error scores were also higher under load (17.68%) than under no-load (2.76%), $F(1, 53) = 91.46$, partial $\eta^2 = 0.633$, and they were higher for benchmarking (11.95%) than for cross-multiplication (6.07%), $F(1, 53) = 16.59$, partial $\eta^2 = 0.238$.

Two interaction effects were also significant: load × load type, $F(2, 52) = 7.98$, partial $\eta^2 = 0.235$, and load × strategy, $F(2, 52) = 4.58$, partial $\eta^2 = 0.150$. Again, this was primarily

Table 1

Median Correct Response Times (in msec) and Combined Error Scores (in %) as a Function of Load Type, Load, and Strategy

		No Load		Load	
Strategy	Measure	M	SE	M	SE
	Phonological Load				
Procedural	RT	2,753	206	2,838	226
	Error	.63	1.1	9.19	1.8
Conceptual	RT	3,162	292	3,131	325
	Error	7.64	2.6	12.54	1.8
	Visuo-spatial Load				
Procedural	RT	2,396	195	2,467	214
	Error	.92	1.0	23.92	1.7
Conceptual	RT	2,891	276	2,462	308
	Error	4.47	2.4	27.86	1.6

due to the pattern of combined error scores. Participants suffered a much higher load penalty in their combined error scores when placed under concurrent visuo-spatial load than they did when placed under phonological load, $F(1, 53) = 16.06$, partial $\eta^2 = 0.233$. Regarding the load × strategy interaction, the load effect on procedural strategies was higher than the load effect on conceptual strategies, $F(1, 53) = 4.23$, partial $\eta^2 = 0.074$.

The strategy × load type interaction was not significant, $F(2, 52) = 0.410$, $p = 0.67$, nor was the three-way interaction of load type, load, and strategy, $F(2, 52) = 0.668$, $p = 0.517$. This indicated that strategy type (procedural / conceptual) is not tied to a specific working memory component. Instead, load effects on specific fraction strategies seem to be load-general.

Strategy Choice

To analyze the effects of working memory load on the choice of strategy used in a fraction comparison, a 2 x 2 analysis of variance was conducted on the percentages of each strategy used with working memory load type (visuo-spatial vs. phonological) as a between-subjects factor and load (load vs. no-load) as a within-subjects factor (see Table 2). There were no effects of load or load type on strategy selection (the highest *F* value was 1.81).

Discussion

The present study found that performance on a fraction comparison task depends on the availability of working memory resources. This was expected given the role that working memory plays in most types of mental arithmetic (DeStefano & LeFevre, 2004). Intriguing findings in this study were the critical interactions of Load × Load-Type and Load × Strategy. Participants under a visuo-spatial load made significantly more errors (relative to the no-load

Table 2

Mean Percentages of Strategy Choice as a Function of Load Type and Load

	No Load		Load	
Strategy	M	SE	M	SE
Phonological Load				
Procedural	71.3	8.1	78.5	10.8
Conceptual	28.7	8.1	21.5	10.8
Visuo-spatial Load				
Procedural	69.4	7.2	57.5	9.8
Conceptual	30.6	7.2	42.5	9.8

condition) than did those participants who were under a phonological load. This interaction effect did not depend on strategy type, indicating a significant role for the visuo-spatial sketchpad in both procedural and conceptual strategies for mental fraction comparison. The phonological load effect was not absent, but it was not as large as the visuo-spatial load effect.

Regarding the Load x Strategy interaction, execution of a procedural strategy suffered more under load than did the execution of a conceptual strategy. This is likely due to the multi-step nature of procedural strategies. Multi-step problems use comparatively more working memory resources than do single-step problems (Ashcraft & Kirk, 2001). The Load x Strategy interaction did not depend on the type of working memory load. It is not immediately clear why this is the case. One may speculate that because all fraction stimuli were composed of single-digit numerators and denominators, the critical role for working memory came at the comparison stage for both strategies. This may imply that both strategies critically involve a magnitude judgment that takes place mostly in the visuo-spatial sketchpad, hence leading to the large visuo-spatial load effect.

The results of the present study provide an important contribution to the overall understanding of adults' numerical and mathematical cognition, but they also have implications in mathematics education regarding the cognitive differences between conceptual and procedural strategies. Future research will need to investigate the visual/spatial distinction in the visuo-spatial sketchpad, and the role of the central executive will also need to be addressed. This future work will add to the overall understanding of the role of working memory in mathematical cognition.

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